Ph217c, Homework 3.
Due Thursday, April 24, 2008.

1. [60%] **Gravitational waves from power-law expansion.** Suppose that the Universe underwent an inflationary epoch with constant $w = p/\rho$ in the range $-1 < w < -1/3$.

(a) Calculate the relation between η and aH in this case.

(b) Write the differential equation for E. By a substitution of the form $E = (-\eta)^q X$ for some q, turn this into the differential equation for a Bessel function.

(c) Using the large-argument oscillatory solution for the Bessel function, relate the cosinelike and sinelike oscillation amplitudes to the final ($\eta = 0$) amplitude $E(0)$.

(d) What is the power spectrum of primordial gravitational waves in this model?

2. [40%] **Exponential potential.**

(a) Determine the slow-roll parameters, primordial power spectra, scalar spectral index n_s, and tensor-to-scalar ratio r for the following potential:

$$V(\phi) = V_0 \exp\left(-\sqrt{\frac{16\pi G}{p}}\,\phi\right). \tag{1}$$

(b) Show that inflation never ends for this potential. Can you think of a way to solve this problem?