Ph217c, Homework 1.
Due Thursday, April 10, 2008.

1. [40%] **Momentum evolution for a massless particle.** In class, it was claimed that to first order in perturbation theory, the spatial momentum of a massless particle varies as:

$$\dot{p} = -\left(aH + \hat{p}^i \frac{\partial A}{\partial x^i} + \hat{p}^i \hat{p}^j \frac{\partial B_i}{\partial x^j} + \dot{D} + \hat{p}^i \hat{p}^j \dot{E}_{ij} \right) p. \qquad (1)$$

Prove this equation.

2. [30%] **Photon perturbation from gravitational waves.** Prove that the gravitational source term for the $l = 2$, $m = \pm 2$ photon multipole moments is:

$$\dot{\Theta}_{2,\pm 2}|_{\rm grav} = \frac{1}{5\sqrt{6}} (\dot{E}_{11} - \dot{E}_{22} \pm 2i\dot{E}_{12}). \qquad (2)$$

3. [30%] **Thomson scattering term.** We showed in class that in multipole space, the contribution to the photon perturbation from scattered radiation was:

$$\dot{\Theta}_{lm}|_{\rm S3} = -\frac{i^l}{\sqrt{4\pi(2l+1)}} \int\int Y^*_{lm}(\hat{\mathbf{p}}) \frac{3}{16\pi} \dot{\tau} [1 + (\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}')^2] \Theta(\hat{\mathbf{p}}')\, d^2\hat{\mathbf{p}}\, d^2\hat{\mathbf{p}}'. \qquad (3)$$

This problem works through the evaluation of the right-hand side.

(a) Decompose the factor $1 + (\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}')^2$ into Legendre polynomials, $P_L(\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}')$. Show that only the $L = 0$ and $L = 2$ terms appear in the sum.

(b) Using the spherical harmonic addition theorem (Eq. C12 of Dodelson, or any advanced quantum mechanics text), re-write Eq. (3) purely in terms of spherical harmonics of $\hat{\mathbf{p}}$ and $\hat{\mathbf{p}}'$.

(c) Show that

$$\dot{\Theta}_{lm}|_{\rm S3} = \begin{cases} -\dot{\tau}\Theta_{00} & l = 0 \\ -\frac{1}{10}\dot{\tau}\Theta_{2m} & l = 2 \\ 0 & \text{otherwise} \end{cases} . \qquad (4)$$