Ph217b, Homework 3.
Due Thursday, March 6, 2008.

1. [40%] **Neutron abundance at freeze-out.** Dodelson #3.4.

2. [20%] **Extra neutrinos.** Suppose that there was a fourth generation of active neutrinos in addition to ν_e, ν_μ, and ν_τ. (This possibility is now excluded by the Z lifetime, but you should still be able to do the problem.)

(a) Compute $g_\star$ prior to e^+e^- annihilation, and $g_{\star,eff}$ after e^+e^- annihilation.

(b) From your solution to problem 1, what is the final neutron abundance in this case? What will this do to the final ^{4}He abundance?

3. [20%] **Carbon (non)-production in BBN.** The isotope ^{12}C is produced in stars by the "triple alpha" process:

$$\begin{aligned} 2\,{}^4\mathrm{He} &\leftrightarrow {}^8\mathrm{Be} \\ {}^8\mathrm{Be} + {}^4\mathrm{He} &\rightarrow {}^{12}\mathrm{C} + \mathrm{photons}. \end{aligned} \tag{1}$$

The rate of carbon production by this mechanism, neglecting electron screening, is given by:

$$\dot{X}_\mathrm{C} = 2 \times 10^{-57} \frac{e^{-4.294/T_9}}{T_9^3} \left(\frac{n_b}{\mathrm{cm}^{-3}}\right)^2 X_\mathrm{He}^3\,\mathrm{s}^{-1}, \tag{2}$$

where T_9 is the temperature in GigaKelvin. Assuming that the ^{4}He abundance is negligible until $T_9 = 0.8$, and is 0.24 thereafter, compute the final abundance of primordial carbon produced by the triple alpha process. (An order-of-magnitude answer will suffice for this problem.)

4. [20%] **Expansion history during matter-radiation transition.** Derive the expression in class for the proper time as a function of $y = a/a_\mathrm{eq}$:

$$t = \frac{2.6\,\mathrm{kyr}}{(\Omega_m h^2)^2} \left[\frac{2}{3}(1+y)^{3/2} - 2(1+y)^{1/2} + \frac{4}{3}\right]. \tag{3}$$

What is the corresponding equation for the conformal time?