Ph217b, Homework 2.
Due Thursday, February 28, 2008.

1. [20%] **Practice with magnitudes.** This problem deals with magnitudes in the visual ("V") waveband. This waveband is centered at a wavelength of λ =5500Å (green) and has a width $\Delta\lambda = 900$Å. The magnitude V in this band is related to the flux F by

$$V = -\frac{5}{2}\log_{10}\frac{F}{(3.631\times 10^{-23}\,\mathrm{W/m^2})(\Delta\nu/\mathrm{Hz})}. \qquad (1)$$

where F is the flux in the band, $\Delta\nu$ is the range of frequencies within the band, and the denominator is a constant that depends on the version of the magnitude system used (this one is known as the "AB" system).

(a) Determine the central frequency of the V band and its width $\Delta\nu$ in Hz.

(b) Suppose that a light bulb emits 2 W in the V band, and that the human eye can see objects as faint as 6th magnitude ($V = 6$) against a black background. What is the maximum distance from which the human eye could see this light bulb?

(c) The absolute magnitude of an object M_V is its magnitude as seen from a distance of 10 parsecs. What is the absolute magnitude of the light bulb in part (b)?

(d) The Sun has an absolute magnitude of $M_V = 5$. If, with a small amateur telescope, you can see objects as faint as $V = 10$, from what distance could you see the Sun? What about a Type Ia supernova with absolute magnitude $M_V = -18$?

2. [20%] **Luminosity distance-redshift relation.** Consider a universe containing only matter and cosmological constant, and possibly with spatial curvature.

(a) Taylor-expand the radial comoving distance $\chi(z)$ to order z^2.

(b) Use this expansion to compute the luminosity distance $D_L(z)$ to order z^2. Show that the first two coefficients depends only on H_0 and the combination

$$q_0 = \frac{1}{2}\Omega_m - \Omega_\Lambda, \qquad (2)$$

known as the "deceleration parameter."

3. [20%] **Peak angular diameter distance.** For the Einstein-de Sitter universe (flat, $\Omega_m = 1$), find the redshift at which the angular diameter distance D_A is maximized. What is the value of D_A?

4. [20%] **Density of the CMB.** Suppose the Hubble constant today is $H_0 = 70$ km/s/Mpc, and the cosmic microwave background is a blackbody at a

temperature of 2.73 Kelvin. What is the energy density of the CMB? What is its density parameter Ω_{cmb}?

5. [20%] **Properties of relativistic plasma.** Prove the following statements for a thermalized relativistic plasma containing noninteracting particles of arbitrary mass and no chemical potential:

(a) The function $g_\star(T)$ is nondecreasing.

(b) The equation of state $w = p/\rho$ satisfies the inequality $0 \leq w \leq 1/3$.