1. **Practice with magnitudes.** This problem deals with magnitudes in the visual ("V") waveband. This waveband is centered at a wavelength of \( \lambda = 5500\text{Å} \) (green) and has a width \( \Delta \lambda = 900\text{Å} \). The magnitude \( V \) in this band is related to the flux \( F \) by

\[
V = -\frac{5}{2} \log_{10} \left( \frac{F}{(3.631 \times 10^{-23}\text{W/m}^2)(\Delta \nu/\text{Hz})} \right),
\]

where \( F \) is the flux in the band, \( \Delta \nu \) is the range of frequencies within the band, and the denominator is a constant that depends on the version of the magnitude system used (this one is known as the "AB" system).

(a) Determine the central frequency of the \( V \) band and its width \( \Delta \nu \) in Hz.

(b) Suppose that a light bulb emits 2 W in the \( V \) band, and that the human eye can see objects as faint as 6th magnitude (\( V = 6 \)) against a black background. What is the maximum distance from which the human eye could see this light bulb?

(c) The absolute magnitude of an object \( M_V \) is its magnitude as seen from a distance of 10 parsecs. What is the absolute magnitude of the light bulb in part (b)?

(d) The Sun has an absolute magnitude of \( M_V = 5 \). If, with a small amateur telescope, you can see objects as faint as \( V = 10 \), from what distance could you see the Sun? What about a Type Ia supernova with absolute magnitude \( M_V = -18 \)?

2. **Luminosity distance-redshift relation.** Consider a universe containing only matter and cosmological constant, and possibly with spatial curvature.

(a) Taylor-expand the radial comoving distance \( \chi(z) \) to order \( z^2 \).

(b) Use this expansion to compute the luminosity distance \( D_L(z) \) to order \( z^2 \). Show that the first two coefficients depends only on \( H_0 \) and the combination

\[
q_0 = \frac{1}{2} \Omega_m - \Omega_\Lambda,
\]

known as the “deceleration parameter.”

3. **Peak angular diameter distance.** For the Einstein-de Sitter universe (flat, \( \Omega_m = 1 \)), find the redshift at which the angular diameter distance \( D_A \) is maximized. What is the value of \( D_A \)?

4. **Density of the CMB.** Suppose the Hubble constant today is \( H_0 = 70\text{ km/s/Mpc} \), and the cosmic microwave background is a blackbody at a
temperature of 2.73 Kelvin. What is the energy density of the CMB? What is its density parameter $\Omega_{\text{cmb}}$?

5. [20%] Properties of relativistic plasma. Prove the following statements for a thermalized relativistic plasma containing noninteracting particles of arbitrary mass and no chemical potential:
   
   (a) The function $g_\nu(T)$ is nondecreasing.
   
   (b) The equation of state $w = p/\rho$ satisfies the inequality $0 \leq w \leq 1/3$. 
