HOMEWORK 1 – Ph217b COSMOLOGY
Due Tuesday **February 19, 2008**.

1. [50%] We considered in class the FRW metric,

$$ds^2 = -dt^2 + a^2(t)[d\chi^2 + f(\chi)(d\theta^2 + \sin^2\theta\, d\phi^2)]. \tag{1}$$

(a) Compute the Christoffel symbols for this metric, for general $f(\chi)$.
(b) Compute the Ricci tensor components $R^t{}_t$ and $R^\chi{}_\chi$.
(c) Relate $R^t{}_t$ and $R^\chi{}_\chi$ to the energy density and pressure measured by a comoving observer.
(d) Argue that for a homogeneous and isotropic universe, $R^\chi{}_\chi$ can only depend on t and not on χ. Use this fact to show that $f(\chi)$ must obey a differential equation

$$\frac{f'^2}{2f^2} - \frac{f''}{f} = \text{constant}. \tag{2}$$

(e) Find the most general solution to this differential equation. (Hint: substitute $f = h^2$.) Show that all solutions are either (i) singular at the origin $\chi = 0$, or (ii) isometric to one of the three solutions considered in class (flat, closed, open).
(f) Derive the Friedmann equations from your solution to part (b) and the value of the constant you found in Eq. (2).

2. [25%] Consider an FRW universe that is empty, i.e. $\rho = p = 0$, and expanding, $H > 0$. (This is called the *Milne universe*.)
(a) Show that this universe must be open, and solve for the scale factor $a(t)$. Use coordinate re-scalings to set $K = -1$ and set the Big Bang to have occurred at $t = 0$.
(b) Show that with the coordinate transformation:

$$\begin{aligned}
x^0 &= t\cosh\chi, \\
x^1 &= t\sinh\chi\cos\theta, \\
x^2 &= t\sinh\chi\sin\theta\cos\phi, \\
x^3 &= t\sinh\chi\sin\theta\sin\phi,
\end{aligned} \tag{3}$$

that the Milne metric is actually a description of Minkowski space with an unusual coordinate system. What does this imply about the Big Bang singularity in the Milne metric?

3. [25%] Suppose that we live in a ΛCDM universe with $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, and $H_0 = 70$ km/s/Mpc. Compute the total density ρ, pressure p, Hubble rate H, age of the Universe t, conformal time η, angular diameter distance D_A, and luminosity distance D_L at redshifts $z = 1$ and $z = 2$. (For the last parts you will need to do a numerical integral.)