Estimating the small-scale galaxy correlation function

Yacine Ali-Haïmoud

November 30, 2009

1 The projected correlation function (Davis & Peebles, 1983)

1.1 Notation

One can measure the two-point correlation function in redshift space $\xi^{\rm obs}(r_p, \pi)$ as a function of two variables, π, the radial separation, and r_p, the transverse separation. The observables are redshifts (due to the Hubble flow and intrinsic velocities) and angular positions on the sky. For $z \ll 1$, we have (with $c = 1$) :

$$\pi = (z_1 - z_2)/H_0 \quad , \quad r_p \equiv (z_1 + z_2)/H_0 \times \tan(\theta_{12}/2) \tag{1}$$

The redshift separation between two galaxies is defined as

$$s \equiv \left(z_1^2 + z_2^2 - 2z_1 z_2 \cos(\theta_{12})\right)^{1/2} / H_0 \tag{2}$$

It approximates the true separation r when peculiar velocities are negligible.
On small scales, where $\Delta z \ll 1$, $\theta_{12} \ll 1$, we have $s \approx \left(\pi^2 + r_p^2\right)^{1/2}$.
By small scales, we refer to $\sim 1 - 10$ Mpc.

1.2 Getting rid of the peculiar velocities

For a given true radial separation y, and total separation $r = (r_p^2 + y^2)^{1/2}$, the measured radial separation π can be decomposed as follows :

$$\pi = y - y\ h(r) + v_{\rm pec}/H_0\ . \tag{3}$$

The first term is due to the Hubble expansion. The second term comes from bulk motions ($h(r) = 0$ if the cluster expands with the general expansion on the scale r, $h(r) = 1$ if the cluster is Virialized, $h(r) > 1$ if the cluster is collapsing). The third term comes from the random peculiar velocities in the cluster along the line of sight.
Given a probability distribution for the peculiar velocities $f(v_{\rm pec})$, the observed correlation function is then obtained by the following convolution of the real space correlation function $\xi(r)$ with f :

$$1 + \xi^{\rm obs}(r_p, \pi) = \int_{-\infty}^{+\infty} [1 + \xi(r)]\, H_0 f(v_{\rm pec}) {\rm d}y \quad , \quad r = (r_p^2 + y^2)^{1/2} \quad , \quad v_{\rm pec} = H_0(\pi - y + y\ h(r)) \tag{4}$$

Note that typically, people use $f(v) \propto {\rm e}^{-\sqrt{2}|v|/\sigma}$.
From this we obtain the projected correlation function :

$$w_p(r_p) \equiv \int_{-\infty}^{+\infty} {\rm d}\pi \xi^{\rm obs}(r_p, \pi) = \int_{-\infty}^{+\infty} {\rm d}y\ \xi\left((r_p^2 + y^2)^{1/2}\right) \tag{5}$$

With this procedure, one therefore gets rid of the effect of the peculiar velocities.
This equation can be inverted to recover $\xi(r)$ form the measured $w_p(r_p)$:

$$\xi(r) = -\frac{1}{\pi} \int_r^{+\infty} {\rm d}r_p w'(r_p)(r_p^2 - r^2)^{-1/2} \tag{6}$$

2 A more robust estimator (Padmanabhan et al., 2007)

2.1 Theory

In practise the integral in w_p cannot be extended to infinity and needs to be cut at a cutoff scale $\pi_{\max} \gg r_p$. This renders the estimator sensitive to poorly measured long wavelength modes. Padmanabhan et al. (2007) suggest a new estimator ω, using high-pass filtering to remove the problematic long wavelength modes. They define

$$\omega(r_s) \equiv 2\pi \int_0^{r_s} \mathrm{d}r_p \; r_p \; G(r_p, r_s) w_p(r_p) \quad , \quad w_p(r_p) \equiv \int_{-\pi_{\max}}^{\pi_{\max}} \mathrm{d}\pi \xi^{\mathrm{obs}}(r_p, \pi) \; , \tag{7}$$

where the filter G has a characteristic scale r_s : $G(r_p, r_s) = r_s^{-3} g(r_p/r_s)$, and is "compensated" , i.e. s.t.

$$\int_0^{r_s} \mathrm{d}r_p \; r_p G(r_p, r_s) = 0 \; . \tag{8}$$

This ensures that the slowly varying modes are canceled out.
$\omega(r_s)$ can also be related to the real space correlation function $\xi(r)$. Note that for a power-law correlation function, one obtains $\omega(r_s) \propto \xi(r_s)$.

2.2 Filter $G(r_p, r_s)$

The family of filters $G(r_p, r_s) = r_s^{-3} g(r_p/r_s)$ is considered, with

$$g(x) = x^{2\alpha}(1 - x^2)^{\beta}(c - x^2) \; , \tag{9}$$

where the constant c is chosen such that the integral of $r_p G(r_p, r_s)$ vanishes.
In practise, they adopt $(\alpha, \beta) = (2, 2)$.

2.3 Practical calculation

ξ^{obs} can be estimated via

$$\xi^{\mathrm{obs}}(r_p, \pi) = \frac{DD(r_p, \pi)}{RR(r_p, \pi)} - 1 \tag{10}$$

Therefore, the estimator ω is given by

$$\omega(r_s) = 2\pi \int_0^{r_s} \mathrm{d}r_p \; r_p G(r_p, r_s) \int_{-\pi_{\max}}^{\pi_{\max}} \mathrm{d}\pi \frac{DD(r_p, \pi)}{RR(r_p, \pi)} \tag{11}$$

The constant disappears because of the insensitivity of G to constant changes.
For small bins $\Delta\pi$, Δr_p, one can write

$$RR(r_p, \pi) = 2\pi r_p^2 \overline{n} N \Phi(r_p, \pi) \Delta \ln r_p \Delta\pi \; , \tag{12}$$

where

$$\Phi(r_p, \pi) \equiv \frac{\int S(\mathbf{x_1}) S(\mathbf{x_1} + \mathbf{s}) \mathrm{d}^3\mathbf{x_1}}{\int S(\mathbf{x_1}) \mathrm{d}^3\mathbf{x_1}} \quad , \quad \mathbf{s} \equiv \mathbf{r_p} + \pi \mathbf{e}_\pi \tag{13}$$

If one takes a binning small enough that DD is either 0 or 1, then one has

$$\omega(r_s) = \sum_{i \in DD, r_{p,i} < r_s, |\pi_i| < \pi_{\max}} \frac{G(r_{p,i}, r_s)}{\overline{n} N \Phi(r_{p,i}, \pi_i)} \; , \tag{14}$$

i.e. the integral is transformed into a simple Riemann sum.

Note : the authors claim that this estimator is insensitive to the integral constraint. My understanding is that the integral constraint is an uncertainty in the multiplicative factor in ξ. The high-pass filtering removes additive constants in ξ. I am not quite sure how it works then.

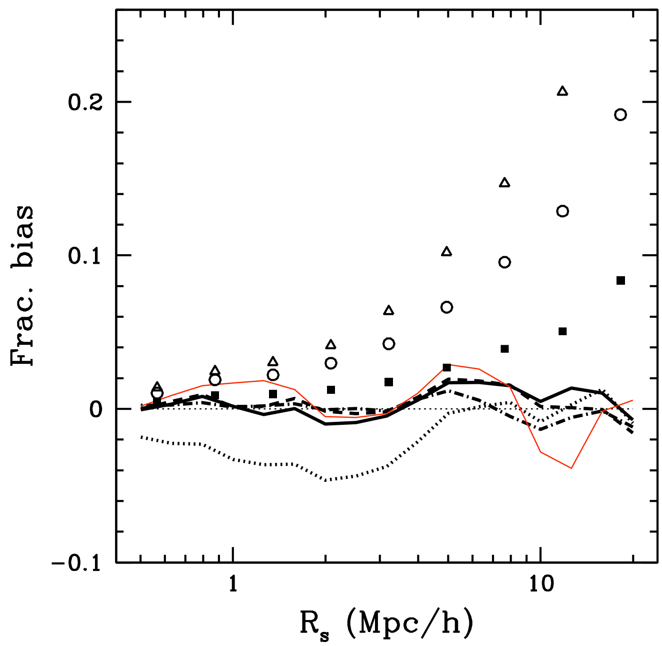

Figure 4. The fractional bias in ω (bias $= (\omega - \omega_{\rm ref})/\omega_{\rm ref}$) obtained from the redshift space distribution of galaxies in the Millennium simulation for different $Z_{\rm max}$ (in Mpc/h) – 20 (dotted), 40 (thick solid), 60 (dashed), and 80 (dot-dashed). The true value (ω_{ref}) is assumed to be obtained by integrating to a $Z_{\rm max} = 100{\rm Mpc}/h$; the figure shows that ω has converged to better than 2% by $Z_{\rm max} = 40{\rm Mpc}/h$. The thin solid (red) line plots ω obtained by integrating over the real space correlation function. The fluctuations in $\Delta\omega$ are consistent with measurement noise. For comparison, the points show the analogous bias terms for w_p, with triangles, circles, and squares corresponding to $Z_{\rm max}$ = 40, 60, and $80{\rm Mpc}/h$ respectively. For a fair comparison with ω, we plot the bias in w_p at $R_s/2$, which roughly corresponds to the central scale probed by ω. Note that the biases in w_p are significantly larger than those for ω.

2.4 Testing the filter with the Millenium simulation

One of the main advantages of ω versus w_p is that it converges faster when one increases $\pi_{\rm max}$. It therefore completes the partial removal of the peculiar velocity distorsions originally intended by w_p. ω is converged to better than 2% by $\pi_{\rm max} = 40h^{-1}$ Mpc. This can be seen their figure 4 reproduced above (where $Z_{\rm max} = \pi_{\rm max}$).

References

Davis & Peebles, 1983, ApJ, 267, 465

Padmanabhan, White & Eisenstein, 2007, MNRAS, 376, 1702