

Spectra of Photoionized Regions

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1. Introduction

Our next task is to consider the emitted spectrum from an H II region. This is one of the most important subjects observationally since spectroscopy provides our principal set of diagnostics on the physics occurring in the region.

This material is (mostly) covered in Chapters 4 and 5 of Osterbrock & Ferland (on which most of these notes are based).

The sources of emission we will consider fall into two categories:

Cooling:

- Infrared fine structure lines
- Optical forbidden lines
- Free-free continuum

Recombination:

- Free-bound continuum
- Recombination lines
- Two-photon continua

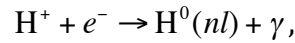
Generally, the forbidden lines (optical or IR) come from metals, whereas the recombination features are proportional to abundance and arise mainly from H and He. Since the cooling radiation has already been discussed, we will focus on recombination and then proceed to diagnostics.

2. Recombination Processes

A. OVERVIEW

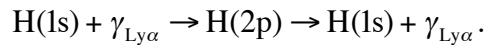
We first consider the case of recombination of hydrogen at very low densities where the collision rates are small, i.e. any hydrogen atom that forms can decay radiatively before any collision. Later on we will introduce collisions between particles (electrons, protons) with excited atoms.

The recombination process at low density begins with a radiative recombination,



which populates an excited state of the hydrogen atom. This is followed by a sequence of radiative decays to successively lower values of n (with l changing by ± 1 at each step). This sequence is subject to the constraint that Lyman lines (decay to $n=1$) are blocked within the assumption of Case B (i.e. lots of neutral hydrogen to re-absorb Lyman series radiation). The sequence of decays thus ends in one of two ways:

- Decay chain ends at 2s: In this case, the atom decays to 1s via emission of two photons.
- Decay chain ends at 2p: In this case, a Lyman- α photon is produced. It begins to resonantly scatter within the nebula, i.e. it is re-absorbed and re-emitted in accordance with:



The resonant scattering process terminates when: (i) the photon diffuses out of the nebula; (ii) the photon receives a Doppler boost in scattering off an H atom on the tail of the Maxwellian (or turbulent) velocity distribution, thereby nudging its frequency far enough from the Lyman- α line center that it can escape; (iii) in the case of an expanding nebula, the photon may redshift out of the line; (iv) the photon is absorbed by dust; (v) the photon is absorbed by a metal atom or ion; (vi) the transient H(2p) atom undergoes a collision or is photoionized. The intensity of emergent Lyman- α radiation depends in detail on whether the ultimate fate of the photon is escape (i,ii,iii) or destruction (iv,v,vi).

An immediate consequence of this argument is that since every recombination results in a decay chain that terminates at $n=2$, exactly one Balmer photon (either line or continuum) is generated in each recombination. This result can be modified in some cases (particularly due to dust), but it is qualitatively useful.

B. LEVEL NETWORKS

To study the quantitative implications of the above process, we introduce the level-resolved recombination coefficient α_{nl} , which is the contribution to α coming from recombinations to the nl level. We further introduce the **cascade matrix** $C(nl, n'l')$, which is the probability that an atom in the nl level will decay via a path that includes $n'l'$. We may also define the **branching probability**,

$$P(nl, n'l') \equiv \frac{A_{nl, n'l'}}{\sum_{n''l''} A_{nl, n''l''}},$$

which is the probability that an atom in the nl level will decay directly to $n'l'$. In the branching probability, we understand that channels blocked by radiative transfer effects (the Lyman lines) are excluded from the denominator.

The rate of production of atoms in the nl level (in $\text{cm}^{-3} \text{s}^{-1}$) is then given by

$$\sum_{n''l''} C(n''l'', nl) \alpha_{n''l''} n_e n_p.$$

The rate of emission of photons in the $nl \rightarrow n'l'$ line is then this rate times the branching probability, and the line emissivity (in $\text{erg cm}^{-3} \text{s}^{-1}$) is obtained by multiplying by $h\nu$:

$$4\pi j_{nl \rightarrow n'l'} = h\nu_{nl \rightarrow n'l'} P(nl, n'l') \sum_{n''l''} C(n''l'', nl) \alpha_{n''l''} n_e n_p.$$

We know how to compute all of the coefficients here except for the cascade matrix. The latter is most easily obtained via induction. It is trivial to see that:

$$C(nl, n'l') = \begin{cases} 0 & n' > n \\ \delta_{ll'} & n' = n \end{cases}.$$

For $n' < n$, we can find the cascade matrix for each $n'l'$ by considering the probability to reach $n'l'$ from any higher level:

$$C(nl, n'l') = \sum_{n''=n'+1}^n \sum_{l''=0}^{n''-1} C(nl, n''l'') P(n''l'', n'l').$$

By this method, we can calculate the hydrogen line ratios in the low-density limit. These in principle depend on temperature, but the dependence is extremely weak, since all of the α_{nl} 's decrease slowly with temperature. For example, at $n_e = 100 \text{ cm}^{-3}$ (see Table 4.4 of Osterbrock & Ferland):

$$j_{\text{H}\alpha}/j_{\text{H}\beta} = 3.04 \text{ (} T = 5000 \text{ K) ... } 2.75 \text{ (} T = 20000 \text{ K)}.$$

In practice, this ratio is often used to estimate dust reddening.

As one might expect, the Balmer lines become successively weaker as we move to higher order:

$$\text{H}\alpha:\text{H}\beta:\text{H}\gamma:\text{H}\delta \approx 2.9:1:0.5:0.25 \quad (T = 10^4 \text{ K}).$$

The Paschen line intensities are also weaker, $j_{\text{Pa}\alpha}/j_{\text{H}\alpha} \approx 0.12$ (although this is in part due to the lower energy per photon).

C. FREE-BOUND CONTINUUM

Continuum radiation is emitted in recombination both via free-bound transitions and two-photon decay. We consider free-bound transitions first. The number of direct recombinations to the n th shell per unit volume per unit time is $\alpha_n n_e n_p$, where

$$\alpha_n = \sum_{l=0}^{n-1} \alpha_{nl}.$$

The emissivity (in $\text{erg cm}^{-3} \text{ s}^{-1} \text{ Hz}^{-1}$) is then

$$j_\nu = h\nu \alpha_n n_e n_p \Pi(\nu),$$

where $\Pi(\nu)$ is the probability distribution for the frequency of the emitted photon. This distribution has a lower cutoff at $3288n^{-2} \text{ THz}$ corresponding to emission from an electron that recombines from zero velocity. It has a typical width of order the thermal energy of an electron, $\Delta\nu \sim kT/h$, and at higher frequencies (more than $\Delta\nu$ above the cutoff) it declines exponentially.

The total intensities $\int j_\nu d\nu$ of the Balmer, Paschen, etc. continua vary with temperature roughly in proportion to the recombination coefficients ($\sim T^{-0.5}$), but their widths $\Delta\nu$ are proportional to T . Thus the value of j_ν contributed by each continuum near its cutoff is proportional to $T^{-1.5}$ (which makes sense since this is contributed by the slowest-moving electrons). This continuum contribution can be measured spectroscopically since one sees a discrete jump in the continuum. The ratio of the continuum jump to the Balmer lines can thus be used to estimate nebular temperature.

D. TWO-PHOTON CONTINUUM

Finally, we come to the hydrogen 2-photon continuum, which arises from the decay:

$$\text{H}(2s) \rightarrow \text{H}(1s) + \gamma + \gamma.$$

The total energy of the two photons is $h\nu_{\text{Ly}\alpha}$. We suppose that the energy is split in the ratio $y:1-y$ with probability $\Pi(y)$. This probability is normalized:

$$\int_0^1 \Pi(y) dy = 2,$$

since there are 2 photons emitted. The rate of 2-photon decays per unit volume per unit time is then:

$$j_v(2\gamma) = h\nu \frac{\Pi(y)}{v_{Ly\alpha}} \sum_{n''l''} C(n''l'', 2s) \alpha_{n''l''} n_e n_p.$$

Typically ~30% of the recombinations end at 2s, so the luminosity in the 2-photon decays is of the same order of magnitude as that in the Balmer lines. The continuum however ranges down to a minimum wavelength of 1216 Å, with half of the photons (and most of the energy) emerging at $\lambda < 2432$ Å. Thus the 2-photon decay can be the dominant UV continuum emission mechanism.

E. COLLISIONS

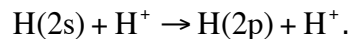
Up until now, we have treated recombination as a process with one proton and one electron that form an atom that then decays in isolation. At high density, it is possible for the atom to undergo collisions before it decays. There are two major circumstances where this is possible:

- The 2s state is metastable, and its unusually long lifetime makes it more likely than other states to suffer a collision before it decays.
- The high- n states are geometrically very large ($\sim a_0 n^2$), and are often long-lived (due to the low frequencies of the photons they emit when they decay).

In the case of 2s, the lifetime is 0.1 s, the typical cross section of an atom is 10^{-15} cm², and the typical velocity of an electron is 10^8 cm/s. Thus we might expect collisions to become important at densities exceeding $\sim 10^8$ cm⁻³. In fact, they become important at densities of $\sim 10^5$ cm⁻³ because the electric field from passing charged particles can induce a **Stark effect**. The 2s and 2p levels are (nearly) degenerate, so any perturbing electric field in (say) the z-direction causes the energy eigenstates to be not 2s and 2p_z but rather:

$$\frac{|2s\rangle \pm |2p_z\rangle}{\sqrt{2}}.$$

The electron then oscillates between the 2s and 2p_z states. The net result of the collision is:



Note that the collisional transition is dominated by protons rather than electrons: the fact that the protons move more slowly enhances their ability to cause oscillations (see next homework).

For the high- n states, the same effect, driven by protons, can cause transitions between different l but the same n . At very high n , collisions with electrons (which produce a time-dependent potential) can change the energy of the atom (hence the value of n). The qualitative effect can be understood as follows. In the capture-cascade picture described in the previous sections, the number density of hydrogen atoms in the nl level is given by:

$$n(nl) = T_{nl} \sum_{n''l''} C(n''l'', nl) \alpha_{n''l''} n_e n_p,$$

where T_{nl} is the lifetime of the nl level. In comparison, at the very high n 's where collisions dominate, we expect the Saha equation to be valid:

$$n_{\text{Saha}}(nl) = (2l+1) \left(\frac{h^2}{2\pi m_e kT} \right)^{3/2} e^{Ry/n^2} n_e n_p.$$

The ratio of the actual abundance to the Saha abundance is called the **departure coefficient** b_{nl} .

The capture-cascade equations usually predict that at large n , $b_{nl} < 1$. Thus at the critical n where collisions begin to dominate – usually of order 100 – the departure coefficient increases with n . In some cases, this can lead to a population inversion and maser activity.

3. Additional Processes

The spectrum of an H II region consists largely of the recombination processes described above, collisionally excited cooling lines, and (in the radio) the free-free continuum. There are, however, a few additional features of the spectrum that should be pointed out.

A. LYMAN- α EMISSION

We need to understand the transport of Lyman- α radiation if we are to understand the emission line strength. The key factor here is the scattering cross section, $\sigma(\nu)$. This can be determined by the principle of detailed balance. We note that in thermal equilibrium at temperature T , the 2p:1s population ratio is $3 \exp(-E_{\text{Ly}\alpha}/kT)$. Therefore, the upward transition rate (in s^{-1}) in a blackbody spectrum must be $3 \exp(-E_{\text{Ly}\alpha}/kT) A_{\text{Ly}\alpha}$ for $E_{\text{Ly}\alpha} \gg kT$.¹ This means that:

$$\int \frac{8\pi\nu^2}{c^2} \sigma(\nu) \exp\left(\frac{-h\nu}{kT}\right) d\nu = 3A_{\text{Ly}\alpha} \exp\left(\frac{-h\nu_{\text{Ly}\alpha}}{kT}\right).$$

This implies that the cross section is:

¹ This restriction is necessary so that we can ignore stimulated emission of Lyman- α .

$$\sigma(\nu) = \frac{3\lambda_{Ly\alpha}^2 A_{Ly\alpha}}{8\pi} \delta(\nu - \nu_{Ly\alpha}).$$

This formula is not directly useful as written because for most applications we need to resolve the width of the Dirac δ -function. This has 3 major sources:

- **Natural broadening:** The excited atom H(2p) has a finite lifetime of ~ 1.6 ns. Therefore its energy must be ill-defined in accordance with the uncertainty principle and the δ -function correspondingly smeared out. A “correct” treatment requires quantum electrodynamics, but the answer can be obtained by treating the wave function of the excited state as a damped harmonic oscillator with damping $\sim \exp(-A_{Ly\alpha}t/2)$. The response of a resonant system with this damping leads to a smearing out of the energy in the oscillator, $\propto [\Delta\omega^2 + (A_{Ly\alpha}/2)^2]^{-1}$. Putting in the normalization suggests:

$$\delta(\Delta\nu = \nu - \nu_{Ly\alpha}) \rightarrow \phi_{\text{nat}}(\Delta\nu) = \frac{A_{Ly\alpha}/4\pi^2}{\Delta\nu^2 + (A_{Ly\alpha}/4\pi)^2},$$

which is correct for an atom at rest.

- **Doppler broadening:** The atoms are not actually at rest, but moving with a Maxwellian velocity distribution. Each component of $\mathbf{v}$ is thus a Gaussian with variance kT/m_H . This implies a distribution of absorption frequencies:

$$\delta(\Delta\nu = \nu - \nu_{Ly\alpha}) \rightarrow \phi_{\text{th}}(\Delta\nu) = \frac{1}{\pi^{1/2}\Delta\nu_D} \exp\left(-\frac{\Delta\nu^2}{\Delta\nu_D^2}\right),$$

where

$$\Delta\nu_D = \sqrt{\frac{2kT}{m_H c^2}} \nu_{Ly\alpha}.$$

- **Turbulent broadening:** If the gas is turbulent then the velocity of the eddies can add to the Doppler broadening. We will neglect it here, which is appropriate for hydrogen if the turbulence is subsonic (recall that the sound speed in an ideal gas is comparable to the speed of the atoms that make it up).

The true broadening $\phi(\Delta\nu)$ is the convolution of both of these, which is called a **Voigt distribution**.

The natural broadening width is $A_{Ly\alpha}/4\pi = 50$ MHz, whereas the Doppler broadening is $106T_4^{1/2}$ GHz. These are both small compared to $\nu_{Ly\alpha} = 2465$ THz. Under ordinary circumstances, the Doppler broadening is larger and dominates. The exception is that the natural broadening has power-law tails to large $|\Delta\nu|$, and so the occasional absorption far from line center is dominated by natural broadening.

In the case where we are dominated by the Doppler broadening, there is a cross section at line center of:

$$\sigma(\nu) = \frac{3\lambda_{\text{Ly}\alpha}^2 A_{\text{Ly}\alpha}}{8\pi^{3/2} \Delta\nu_D} = 6 \times 10^{-14} T_4^{-1/2} \text{ cm}^2.$$

This is ~ 4 orders of magnitude larger than the photoionization optical depth at threshold, so we expect a typical H II region to have an optical depth in Lyman- α of $\tau_{\text{Ly}\alpha} \approx 10^4$. Thus we confirm our expectation that a Lyman- α photon will scatter in the nebula many times (as will other low-order Lyman lines, although it is possible that Lyman- π et al will escape).

It is important to note that for *Galactic* objects, the intervening neutral ISM scatters Lyman- α photons out of the line of sight. Such objects can be observed if they have high-velocity Lyman- α emission (Doppler-shifted away from the absorption). Thus most of our interest in the appearance of Lyman- α radiation is associated with extragalactic (redshifted) objects or with ISM absorption itself.

In the presence of such a large optical depth, one can imagine several ways to lose the photon. In principle, it could random-walk through the nebula to the exterior. Since the mean free path is 10^{-4} of the size of the nebula, roughly $\sim 10^8$ scatterings would be required to random walk to the edge. When it gets there, of course, there is a neutral region so the photon still does not escape. Rather the photon random-walks in *frequency space*, i.e. its frequency ν is redistributed at each scattering. There is a probability of order 10^{-4} that a given scattering occurs off a fast-moving atom that kicks the photon out to a frequency offset with $\exp(-\Delta\nu^2/\Delta\nu_D^2) \sim 10^{-4}$. Then the photon sees an optical depth of only $\tau \sim 1$ and escapes the nebula. The frequency at which the photon emerges is

$$\nu = \nu_{\text{Ly}\alpha} \pm \Delta\nu_D \ln \sqrt{\tau_{\text{Ly}\alpha}}.$$

This implies a double-peaked line profile. This is observed in the spectrum of some extragalactic objects².

An ionized gas that is undergoing expansion (e.g. a stellar wind, the early Universe, ...) can undergo additional processes due to the redshifting of the Lyman- α line, which opens up another escape channel. For an optically thick medium, expanding with a velocity field $\mathbf{v} = H\mathbf{x}$, a photon escapes after scattering $\sim \tau_S$ times, where τ_S is the **Sobolev optical depth**:

$$\tau_S = \frac{3n(\text{H}^0)\lambda_{\text{Ly}\alpha}^3 A_{\text{Ly}\alpha}}{8\pi H}.$$

(This can be derived by integrating the cross section times H^0 density over distance travelled, where “distance” s is now related to frequency by $d\nu/ds = -H/\lambda$.)

² See e.g. the $z=2.8$ Lyman- α emission object that is proposed to be due to recombination radiation from a nearby quasar (Adelberger et al 2006, ApJ 637, 74).

A key feature of expanding models is that the radiation is trapped by multiple scatterings until it is on the red side of the line, at which point it is released and propagates freely. The geometry and velocity structure of the system are clearly of critical importance in modeling the shape of the Lyman- α line.

B. HELIUM PROCESSES

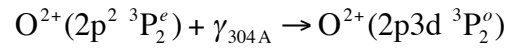
Helium undergoes similar radiative transfer processes in its main resonance lines (He II 304 Å; He I 584 Å). These are not directly observable because of H I absorption in the intervening neutral ISM.

A more interesting case is that the metastable triplet helium level (lifetime 2 hours) can develop a large population, thereby enabling radiative transfer physics in the lines that connect to it (10830 Å, 3889 Å). By partially blocking decays to He I $2^3S_1^o$, this can redistribute the He I triplet recombination line intensities. It can also result in collisional excitation out of He I $2^3S_1^o$ followed by line emission.

C. RESONANCE FLUORESCENCE

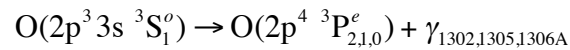
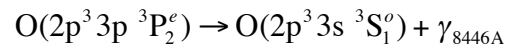
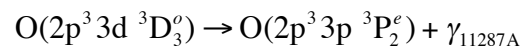
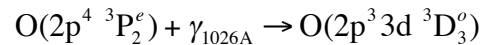
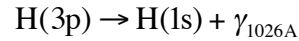
A final process that can affect the spectrum of H II regions is **resonance fluorescence** – the excitation of one species by line radiation from another, followed by a decay sequence that emits lower-energy photons. Two important examples are:

O III:



[20 fluorescence lines at 2809 ... 3811 Å]

O I:



Both of these processes can lead to emission in metal lines that are too weak to observe in recombination and have excitation energies that are too high for collisions.

4. Diagnostics

We are now in a position to consider diagnostics for the temperature and density of H II regions. We have already described the theory, so we will mostly list the basic ideas behind each diagnostic here.

A. TEMPERATURE

Diagnostics for the temperature include:

- *Forbidden lines of np^2 and np^4 ions:* These ions, [N II], [O III], [Ne III] and [S III], have a first excited state that emits a doublet (e.g. [O III] λ 4959,5007Å) and a higher excited state that emits a single line (e.g. [O III] λ 4363Å). There is a rather substantial energy difference, so the ratio of the single to the double line reflects the Boltzmann suppression in the collisional excitation rate (i.e. it increases with temperature). This method has the advantage of using a single species and hence being insensitive to the ionization stage or abundance of the metal. At high densities, it may suffer problems because the doublets are longer-lived and hence more easily suppressed by collisions (lower n_{cr}); an estimate of the density (coming next!) is thus also required. The single line may also be dominated by the hottest part of the nebula (due to its Boltzmann factor), hence one may overestimate the mean temperature.
- *The Balmer continuum:* As noted previously, the jump in the continuum at 3650Å is due to free-bound emission as H^+ ions recombine to $H(n=2)$. The strength of the jump relative to the Balmer lines increases as we decrease T because more of the recombinations to $n=2$ come from very low-energy electrons.
- *Radio continuum:* At low frequencies, the radio spectrum due to free-free emission scales as $I_\nu \propto \nu^{-0.15}$ (it is actually logarithmic in ν – see homework). However, at low frequencies (~ 100 MHz) the emitted radiation exceeds the blackbody spectrum at temperature T ($\propto \nu^2$). The nebula then begins to absorb its own radiation and appear as a blackbody.

B. DENSITY

Diagnostics for the density include:

- *Optical emission line ratios:* Comparison of emission lines of the same ion and similar excitation energies but different critical density can be used to test for the electron density n_e . Of particular use are the doublets where the excited level is fine structure split, e.g. [O II] 3726,3729Å.
- *Fine structure lines:* The ratio of IR to optical lines of the same ion (e.g. O III) can be used in a similar way. The optical/IR line ratios typically increase with temperature (Boltzmann again!) and density (since the fine structure excitations are longer-lived and thus have a lower n_{cr}). Measurement of multiple ratios can enable both T and n_e to be computed.
- *Radio recombination lines:* Transitions such as $H(n=110) \rightarrow H(n=109)$ give rise to lines in the radio part of the spectrum. If maser effects are small, their emissivity is proportional to $n_e n_p$ times combinations of the departure coefficients (with complicated dependence on T and n_e). A fit to many lines can solve for both T and n_e .

C. COMPOSITION

Several diagnostics are available to determine elemental abundances in a nebula.

- *Recombination lines*: The He/H ratio has been determined in many nebulae from the ratio of He I/H I recombination lines. This requires modeling of the temperature structure (for recombination coefficients), the ionization structure (e.g. if there is an H⁺/He⁰ region; He⁰ is unobservable but one may use e.g. the S II/III ratio which is likely similar due to the similar ionization potential), and radiative transfer effects.
- *Forbidden lines*: The abundant metals N, O, Ne, and S have optical cooling lines. The use of their strengths as abundance indicators is dangerous since they are linearly proportional to abundance and exponentially dependent on temperature. In practice one fits a dispersion in the temperature σ_T as well as a temperature.

D. STAR FORMATION RATES

One common application of nebular physics in extragalactic astronomy is measurement of the **star formation rate** (SFR) of a galaxy (Units: $M_{\odot} \text{ yr}^{-1}$). Understanding the history and distribution of cosmic star formation is a hot topic, and many methods of determination have been proposed. They all are based on measuring the most massive stars (with cosmologically short lifetimes), either directly or indirectly. In particular, one could measure:

Direct light from massive stars:
Ultraviolet continuum

Light reprocessed via emission nebulae:
Recombination lines (H α , Pa α)
Forbidden lines ([O II])
Free-free radiation

Light reprocessed by dust:
Far infrared continuum
Mid-infrared emission bands

Effects of supernovae:
Synchrotron radiation (radio)

None of these is perfect (and the debate over which is “best” sometimes extends beyond the realm of facts), and all assume that the **initial mass function** (IMF) of stars – i.e. the distribution of masses of stars that are formed – is known (from studies in our own Galaxy). Our purpose here is to discuss the methods based on nebular physics:³

- *Recombination lines*: Here the idea is fairly simple: since ionizing photons come only from young stars, if we assume a given IMF and use stellar evolution, we conclude

³ The numbers in this section are from Kennicutt, ARA&A 36, 189 (1998).

that each solar mass of stars formed generates 3×10^{60} ionizing photons and hence 3×10^{60} recombinations. If we then take the luminosity in $H\alpha$ (or $Pa\alpha$, etc.) and assume a typical nebular temperature (to correct for the fact that not all recombinations yield an $H\alpha$), we can extract the SFR. The most important drawback here is the effect of dust extinction (see next lecture). $Pa\alpha$ is less sensitive to dust than $H\alpha$, but being in the IR it is much more difficult to measure and hence has not come into common use. A second drawback is the possibility that the ionized region may break out of the galaxy and “leak” ionizing photons into intergalactic space, where they will not contribute to the $H\alpha$ luminosity.

- *The [O II] luminosity:* This is a nebular line, like $H\alpha$. However it arises from cooling, and thus its relation to the flux of ionizing photons is much more complicated (depending on the availability of other coolants, etc.). Moreover being at shorter wavelength (3726, 3729 Å) it is much more sensitive to dust than $H\alpha$. Its chief advantage is being in the optical (i.e. easily observed from the ground) at redshifts out to $z \sim 1.6$ where $H\alpha$ observations would have to be done in the IR.
- *Free-free radiation:* In principle working in the $\lambda \sim 1$ cm band should enable one to observe ionized nebulae irrespective of foreground dust obscuration. The free-free radiation is n_e^2 -weighted and hence its intensity should scale in the same way as recombination lines. It has not proved to be practical as a SFR indicator, largely because when considering whole galaxies, other sources of emission (synchrotron and dust) strongly contaminate it.

In summary, the common SFR indicators from emission nebulae are the $H\alpha$ and [O II] lines.