Interstellar Shocks

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1. Introduction

Having examined the equations of hydrodynamics, we will now study one of their most important applications: **shocks**, i.e. discontinuities in fluid flow. We begin with a discussion of shocks in ideal gases, and then proceed to consider the evolution of a supernova remnant. We conclude by discussing non-ideal behavior in gases and its importance in controlling the behavior of real shocks in the ISM.

References:

Osterbrock & Ferland, Ch. 12
Dopita & Sutherland, Ch. 8

2. Hydrodynamic Shocks

Here we consider the basic equations governing a shock, and in particular what circumstances can result in their formation.

A. ENERGY-CONSERVING SHOCKS

We first consider a simple plane-parallel shock in an ideal gas. We will work in the frame where the shock is stationary. Gas flows into the shock with density ρ_1, velocity v_1, and temperature T_1. It flows out with density ρ_2, velocity v_2, and temperature T_2. All velocities are along the z-axis. Our job is to relate the "1" quantities to the "2" quantities.

The basic approach is to use the conservation laws derived in the lecture on hydrodynamics. The partial derivatives with respect to t, x, and y all vanish because we are moving with the shock.

The first equation is that of mass conservation, which reads:

$$\nabla \cdot (\rho \mathbf{v}) = 0 \to \frac{\partial}{\partial z}(\rho v) = 0.$$

(We drop the z subscript on the velocity.) This tells us very simply that:

$$\rho_1 v_1 = \rho_2 v_2.$$

In other words, the mass flux into the shock equals the mass flux out.

The second equation is that of momentum conservation, which reads:

$$F - v\frac{\partial}{\partial z}(\rho v) - \rho v \frac{\partial v}{\partial z} - \frac{\partial P}{\partial z} = 0.$$

We will assume here that no external forces are applied, so the first term vanishes. We already know that the second term vanishes. Finally, ρv is a constant (see above), so we conclude that:

$$\frac{\partial}{\partial z}(P + \rho v^2) = 0.$$

Thus the momentum flux $P+\rho v^2$ is the same on both sides of the shock. Since the pressure is $\rho kT/\mu$, we find that:

$$\rho_1\left(\frac{kT_1}{\mu} + v_1^2\right) = \rho_2\left(\frac{kT_2}{\mu} + v_2^2\right).$$

It is convenient here to eliminate the density by dividing by ρv:

$$\frac{kT_1}{\mu v_1} + v_1 = \frac{kT_2}{\mu v_2} + v_2.$$

The last equation we will need to close the system is the conservation of energy. We use it in the form of requiring equal stagnation enthalpy on both sides of the shock, i.e. $h_{0,1} = h_{0,2}$:

$$\frac{5kT_1}{2\mu} + \frac{v_1^2}{2} = \frac{5kT_2}{2\mu} + \frac{v_2^2}{2}.$$

These are two equations for two unknowns: T_2 and v_2. We can solve for v_2 if we use the first equation to eliminate T_2:

$$\frac{5kT_1}{2\mu} + \frac{v_1^2}{2} = \frac{5}{2}\left(\frac{kT_1}{\mu}\frac{v_2}{v_1} + v_1 v_2 - v_2^2\right) + \frac{v_2^2}{2}.$$

This is a quadratic equation for v_2. Re-arranging it gives the explicit form:

$$2v_2^2 - \frac{5}{2}\left(\frac{kT_1}{\mu v_1} + v_1\right)v_2 + \frac{5kT_1}{2\mu} + \frac{v_1^2}{2} = 0.$$

This equation has two solutions. One solution must be $v_2=v_1$, since this trivially satisfies the conservation laws. Using the fact that the sum of the two solutions to a quadratic equation is $-b/a$ (where $ax^2+bx+c=0$), we conclude that the nontrivial solution, which describes an actual discontinuity, is

$$v_2 = \frac{5kT_1}{4\mu v_1} + \frac{v_1}{4} = \frac{3c_{s,1}^2}{4v_1} + \frac{v_1}{4},$$

where $c_{s,1}$ is the adiabatic sound speed in the upstream gas. We define the **upstream Mach number** by $M_1 = v_1/c_{s,1}$, so that:

$$v_2 = \left(\frac{3}{4M_1^2} + \frac{1}{4}\right)v_1.$$

The implied density and temperature in the downstream gas are then:

$$\rho_2 = \frac{4\rho_1}{3M_1^{-2} + 1}$$

and

$$T_2 = \left(\frac{7}{8} + \frac{5M_1^2}{16} - \frac{3}{16M_1^2}\right)T_1.$$

We have written down the equations for conservation of mass, momentum, and energy. However, in order to be physical, whatever process occurs to the gas within the shock must accord with the second law of thermodynamics. In particular, the entropy of the gas increases if $T\rho^{-2/3}$ increases, and it decreases (unphysical) if $T\rho^{-2/3}$ decreases. An actual computation shows that this requires $M_1 \geq 1$. If $M_1=1$, then $v_2=v_1$ (repeated root of the quadratic); there is no actual discontinuity in this case. Thus we only need to consider the case that $M_1>1$.

It is instructive to consider two limiting cases here. If $M_1=1+\varepsilon$ is slightly greater than 1, then

$$\frac{v_2}{v_1} = 1 - \tfrac{3}{2}\varepsilon,$$

$$\frac{\rho_2}{\rho_1} = 1 + \tfrac{3}{2}\varepsilon,$$

$$\frac{T_2}{T_1} = 1 + \varepsilon.$$

The discontinuity is extremely small and propagates at (almost) the sound speed. It can be thought of as a sound wave whose waveform is a step function.

The opposite case, which leads to the most dramatic results, is a **strong shock** with $M_1>>1$. In this case, we have $v_2 = v_1/4$, $\rho_2=4\rho_1$, and $T_2 = (5/16)M_1^2T_1$. The gas becomes much hotter, but is only compressed by a factor of 4. This is because the kinetic energy of the incoming gas is randomized and converted to pressure, which prevents further compression of the gas. Larger compression factors are possible if the gas passes through multiple shocks, or can cool, thereby removing the pressure support.

An alternative expression for the temperature after a strong shock is:

$$T_2 = \frac{3\mu v_1^2}{16k} = 1.1\times 10^5 v_7^2 \text{ K},$$

where the numerical value is for ionized hydrogen ($\mu=m_H/2$) and v_7 is the shock velocity in units of 100 km/s.

B. POST-SHOCK COOLING

The above analysis assumes that $Q=0$. This will be true in many cases *at* the shock, but if the cooling time is short compared to the timescale for the shock to cross an object then there are two important consequences. First, the postshock gas can cool, so that T_2 may be far below that given by the above formula. Second, the cooling radiation can have significant effects, e.g. by ionizing the upstream gas before it reaches the shock. We consider the hydrodynamic effects here.

If we consider the "shock" to include not just the hydrodynamic discontinuity but also the cooling zone behind it, then T_2 should be determined by thermal equilibrium considerations instead of by the energy equation. In this case, the momentum equation

$$\frac{kT_1}{\mu v_1} + v_1 = \frac{kT_2}{\mu v_2} + v_2$$

can be solved directly for v_2. In general, if the shock is highly supersonic in the upstream medium, then the second term on the left-hand side dominates. There are then two solutions for v_2, a fast unphysical solution and a slow physical solution where the first term dominates:

$$v_2 = \frac{kT_2}{\mu v_1}.$$

The post-shock density and thermal pressure are:

$$\rho_2 = \frac{\mu \rho_1 v_1^2}{kT_2} \quad \text{and} \quad P_2 = \rho_1 v_1^2.$$

As one can see, the compression ratio ρ_2/ρ_1 can be very large for fast shocks. The post-shock velocity in the shock frame v_2 becomes very small. Thus if one looks at the problem in the rest frame of the upstream gas, the shock sweeps by, picking up the gas and compressing it into a thin shell that moves along with the shock.

3. Supernova Remnants

It is now time for us to take the tools we have developed and apply them to the problem of large explosions and their effects on the ISM. The best-studied such problem is that of a **supernova remnant** (SNR). This is the object generated when a star explodes, dumping some amount of mass M_* outward at some initial velocity v_i. The total *kinetic* energy released by the supernova is

$$E_{SN} = \frac{1}{2} M_* v_i^2.$$

Typical values are $M_* \sim 10^{34}$ g and $v_i \sim 10^9$ cm/s.

A supernova remnant goes through several phases. In the first several weeks of the expansion, the supernova ejecta undergoes **free expansion** – i.e. the gas moves outward at constant velocity. The expansion of the gas causes cooling, but the gas remains hot due to the decay of radioactive isotopes (e.g. ^{56}Ni) produced in the core of the progenitor star. The gas is opaque at this time, even to γ-rays, and so it captures the radioactive energy and glows like an incandescent light bulb; this is the phase that we "see" as the supernova. As the remnant continues to expand, it becomes optically thin and the ^{56}Ni decays; it thus fades from view in the optical, but it continues to emit γ-rays from longer-lived isotopes.

Our main focus here will be on the interaction of the ejected gas with the ISM. Gas present in the ISM must initially get displaced by the ejecta, which is typically moving at ~10^4 km/s, i.e. highly supersonic. Thus there is a shock that propagates outward into the ISM, called the **blast wave**, which accelerates ISM material away from the supernova. Similarly, material ejected from the supernova must get slowed down since it is transferring momentum to the ISM gas. Since the supernova ejecta also becomes cold via expansion (in the sense that its sound speed is small compared to the expansion velocity) this slowdown also occurs via a shock. This shock, known as the **reverse shock**, propagates inward in the frame of the ejected gas (but outward in the lab frame). Between the blast wave and the reverse shock lies a **contact discontinuity**, which separates the shocked supernova ejecta and shocked ISM. The contact discontinuity is not a shock: since the material on both sides moves subsonically (both have already been shock-heated), the contact discontinuity is a surface of pressure equilibrium between material of different composition and entropy.

As the SNR continues to age, most of its material crosses the reverse shock. Eventually, after it has swept up $>M_*$ in mass, the blast wave is left to propagate outward through the ISM, carrying the supernova's energy but leaving the ejecta behind. The blast wave is pushed forward by the thermal pressure of the ball of shock-heated gas behind it. This is called the **Sedov-Taylor phase** (after the physicists who solved the problem of an expanding blast wave in the Earth's atmosphere caused by the explosion of an atomic bomb). Finally, when the slower blast wave and longer timescale allow the swept-up ISM to cool, the SNR enters a **snowplow phase**, in which the results of §IIB apply and the swept-up gas forms a thin shell. The snowplow may be subject to instabilities that give even an initially spherical SNR a complex appearance.

We now consider each of these phases in turn.

A. EARLY EVOLUTION

A SNR is initially in free expansion. This phase lasts until the SNR has swept up its own mass in material in the ISM. We may find the radius at which this transition occurs. If we set the SNR's radius to encompass mass M_*, then we find:

$$\tfrac{4}{3}\pi R^3 m_{\rm H} n_0 = M_*,$$

where n_0 is the initial density of the ambient medium in nucleons per cm^3, or

$$R = 2.1 n_0^{-1/3}\left(\frac{M_*}{M_{\rm Sun}}\right)^{1/3} \text{pc}.$$

The time required is $R/v_{\rm i}$ or:

$$t = 200 n_0^{-1/3} v_9^{-1}\left(\frac{M_*}{M_{\rm Sun}}\right)^{1/3} \text{yr}.$$

Thus e.g. SN 1987A is still mostly in free expansion.

B. SEDOV-TAYLOR PHASE

Our next task is to consider the phase where the blast wave has swept up more mass than was initially present in the ejecta. This phase can be treated in a self-similar way: while the SNR grows with time, and hence has nontrivial dependence on both r and t, the system at a later time is simply a scaled-up version of the system at an earlier time.

Dimensional analysis suggests a particular arrangement of times and radii. Once M_* is negligible compared to the swept-up mass, and if the initial gas is presumed cold ($T_1 \rightarrow 0$), the only parameters involved are the initial energy E_{SN} and the initial density $\rho_0 = m_H n_0$. The only dimensionless function of these variables is:

$$\xi = \frac{r}{(E_{\rm SN}/\rho_0)^{1/5} t^{2/5}}.$$

Therefore all flow variables must be functions of ξ times the appropriate dimensionful quantities constructed out of E_{SN}, ρ_0, and t. In particular, the density, velocity, and sound speed are:

$$\rho = \rho_0 D(\xi),\ \ v = \left(\frac{E_{\rm SN}}{\rho_0}\right)^{1/5} t^{-3/5} u(\xi),\ \ c_s = \left(\frac{E_{\rm SN}}{\rho_0}\right)^{1/5} t^{-3/5} s(\xi).$$

The blast wave is at some radius ξ_s that we wish to solve for. The shock velocity relative to the upstream undisturbed ISM ($\xi > \xi_s$) is then:

$$v_1 = \frac{dr_s}{dt} = \frac{2}{5}\xi_s \left(\frac{E_{\rm SN}}{\rho_0}\right)^{1/5} t^{-3/5},$$

and from this we may obtain the boundary conditions at $\xi = \xi_s$ on the inside of the blast wave. The density jumps by a factor of 4 for a strong shock, so:

$$D(\xi_s^-) = 4.$$

The velocity in the shock frame is slowed to ¼ of its original value, so the post-shock velocity in the lab frame (which is the same as the frame of the undisturbed gas) is ¾ of v_1; thus:

$$u(\xi_s^-) = \frac{3}{4}\left(\frac{2}{5}\xi_s\right) = \frac{3}{10}\xi_s.$$

The sound speed of the post-shock gas for a strong shock is $c_s^2=(5/16)v_1^2$, so we find:

$$s(\xi_s^-) = \frac{\sqrt{5}}{4}\left(\frac{2}{5}\xi_s\right) = \frac{1}{2\sqrt{5}}\xi_s.$$

The evolution inside the blast zone requires solving a system of PDEs (that can be converted to ODEs by use of self-similarity). In principle, one could then solve for ξ_s by requiring the total energy in the interior to be E_{SN}. However, this is too complicated to solve in class. We may obtain an approximate value by assuming that all of the swept-up material moves at velocity $v(\xi_s^-)$ and has the sound speed given by $s(\xi_s^-)$. This gives a total energy of:

$$E_{\rm SN} \approx \frac{4}{3}\pi\rho_0\left[\frac{E_{\rm SN}^{1/5}t^{2/5}}{\rho_0^{1/5}}\right]^3\left[\frac{1}{2}\left(\frac{E_{\rm SN}^{1/5}}{\rho_0^{1/5}t^{3/5}}u(\xi_s^-)\right)^2 + \frac{9}{10}\left(\frac{E_{\rm SN}^{1/5}}{\rho_0^{1/5}t^{3/5}}s(\xi_s^-)\right)^2\right].$$

(The first term in brackets is the kinetic energy and the second is the thermal energy.) This reduces to:

$$1 \approx \frac{4}{3}\pi\xi_s^5\left(\frac{9}{100}\right),$$

or $\xi_s \approx 1.2$. The implied blast wave radius is:

$$R = 1.2E_{\rm SN}^{1/5}\rho_0^{-1/5}t^{2/5} = 6E_{\rm SN,51}^{1/5}n_0^{-1/5}t_{\rm kyr}^{2/5}\ {\rm pc}.$$

Its velocity is

$$v_1 = 0.5E_{\rm SN}^{1/5}\rho_0^{-1/5}t^{-3/5} = 2000E_{\rm SN,51}^{1/5}n_0^{-1/5}t_{\rm kyr}^{2/5}\ {\rm km/s}.$$

The temperature immediately behind the blast wave is:

$$T_2 = 0.16\frac{\mu}{k}E_{\rm SN}^{2/5}\rho_0^{-2/5}t^{-6/5} = 9\times 10^7 E_{\rm SN,51}^{1/5}n_0^{-1/5}t_{\rm kyr}^{2/5}\ {\rm K}.$$

The more detailed treatment (see O&F Ch. 12) gives slightly smaller R, v_1, and T_2 (by up to 40% for T_2).

C. POST-SEDOV-TAYLOR PHASE

The Sedov-Taylor phase results in a shell of shocked interstellar gas whose density is ~4 times the initial density (with a rarefaction in the center) and whose temperature starts high but falls as $t^{-6/5}$. In the center of this is a bubble of hot, shocked ejecta. The shocked ISM however contains a fraction of order unity of the energy.

The cooling time of the shocked ISM decreases as it ages. Therefore, a time will come at which the cooling time is equal to the SNR age. At this point, the hot, low-density ejecta is still unable to cool, but the shocked ISM cools efficiently and is thus compressed into a thin shell. The blast wave is then pushed forward by the pressure of the internal hot bubble. The energy content of the bubble is described by adiabatic evolution,

$$E_{\rm bubble} \propto T_{\rm bubble} \propto \rho_{\rm bubble}^{2/3} \propto R^{-2}.$$

However, we already learned from dimensional analysis that the radius of a bubble is proportional to $E^{1/5}t^{2/5}$. The combination of these proportionalities gives:

$$R \propto (R^{-2})^{1/5} t^{2/5} \rightarrow R \propto t^{2/7}.$$

Thus the power-law exponent of the expansion switches from 2/5 to 2/7.

The hot bubble cools and its density drops as it expands, $\rho \sim t^{-6/7}$ and $T \sim t^{-4/7}$. So long as the cooling function $\Lambda(T) \sim T^{\alpha}$ with $\alpha<¼$ (which occurs in practice due to metal line cooling at intermediate temperatures), the ratio of age to cooling time $\sim t\rho\Lambda$ will increase and eventually the hot bubble will cool. Thereafter the SNR enters the snowplow phase. The snowplow is distinguished by the inability to transport momentum through the SNR because the internal thermal pressure is lost. In this case, the expansion is governed by local momentum conservation as the ejecta undergoes a completely inelastic collision with the surrounding ISM. The momentum conservation rule tells us that:

$$\rho_0 R^3 \dot{R} = \text{constant} \quad \rightarrow \quad R \propto t^{1/4}.$$

This completes the description of the idealized phases of SNR expansion.

D. INSTABILITIES

The contact discontinuity may undergo a **Rayleigh-Taylor instability**. This is because the inward acceleration of the discontinuity ($d^2r_{\rm d}/dt^2<0$) implies that an observer standing at the discontinuity sees a fictitious "gravitational" force outward. This observer thus sees supernova ejecta sitting on top of (and being supported by) shocked ISM material. If the shocked ejecta is denser than the shocked ISM, then such a situation is unstable (much like trying to support a layer of mercury on top of a bath full of water).

Additional deviations from spherical symmetry occur if the supernova ejecta is clumpy or the SNR is expanding into a clumpy medium. In this case, the shocks (reverse or forward) are temporarily stalled by the clumpy material, but propagate freely into the underdense regions.

4. Advanced Shock Physics

We now consider several more advanced topics associated with shocks, at a qualitative level.

A. RADIATION FROM SHOCKS

Radiation from shocks is a complex subject. One must treat both the hydrodynamics, as well as the collisional ionization and cooling of the gas. Furthermore, the cooling radiation contains extreme-UV/X-ray radiation that can cause photoionization, including photoionization before shock passage since light travels faster than shocks. Iterative computational models are required.

A few basic parameters can be obtained however. First is the cooling time of the postshock gas. If the cooling function $\Lambda \sim T^{-1.2}$, as is typical at $T<10^7$ K due to metal cooling, then we can find the cooling time:

$$\tau_{\rm cool} \sim 200 \frac{v_7^{4.4}}{Z n_0} \text{ yr},$$

where Z is the metallicity. We can thus see that young SNRs (e.g. SN 1987A) are unlikely to have undergone any significant cooling of shocked material.

The emission lines from shocks are characterized by much higher temperature to ionization energy ratios than photoionized H II regions, since in shocks the thermal energy is the source for the ionization energy. Therefore, shock-heated gas should be expected to have much higher [O III] 4363/5007Å ratios than photoionized gas. Moreover, the ultraviolet allowed lines, e.g. C IV 1548,1551Å, can be excited. A second feature is that the post-shock gas must recombine gradually, leading to a much larger partially ionized zone (mixed H^+/H^0) than in regions photoionized by stars. This dramatically enhances the emission in lines such as [O I] 6300Å, since O^0 can only exist in H^0 zones but requires some ionization so that it can be excited by electron impact.

B. BALMER-DOMINATED SHOCKS

The above discussion presumes that one sees the cooling zone behind the shock. In some cases, however, the shock is not old enough to have a fully developed cooling zone. In this case, netural H will stream into the shock region, resulting in a so-called **Balmer-dominated shock**. This is characterized by very strong Balmer lines relative to metal lines. The Balmer lines have both a broad and narrow component.

The shock occurs first in the ion-electron plasma, which rapidly randomizes its energy through plasma interactions.[1] The neutrals are affected only by collisions, and propagate into the post-shock plasma where they are ionized.

The narrow component of the H lines is produced by neutral H atoms that cross the shock. They subsequently find themselves in a hot environment, and are collisionally excited and emit line radiation. This component is narrow because large impact parameter collisions can effectively excite the electron into a higher energy level, but tend to be less efficient at deflecting the protons; so this component retains the initial velocity of the incoming gas. The broad component, in contrast, comes from hot ions that undergo charge exchange:

$$H^+ + H \rightarrow H + H^+.$$

The exchange process involves only the hopping of an electron, and thus enables the production of a population of "hot" H atoms (which are eventually collisionally ionized).

The Balmer-dominated shock requires the existence of at least a partially neutral medium into which the shock propagates (in this case the supernova ejecta, which recombined when it was young, small, and dense). As more material is shock-heated and the SNR is bathed in its own thermal EUV/X-ray radiation, an ionization front can be launched into the ejecta. This seems likely to happen to SN 1987A within the next several decades, and the Balmer-dominated shock will be destroyed, replaced by a shock propagating into fully ionized material.

C. MAGNETIC FIELDS

Shocks can also propagate in magnetized gas. While the general analysis is quite complicated, we will consider here the special case of a shock propagating in the z direction with a magnetic field purely in the x direction. We further assume the medium to be magnetic pressure dominated, i.e. the gas pressure will be neglected. This is likely to be appropriate if the post-shock gas can cool, thereby removing its thermal pressure.

The basic equations are as follows. For the mass conservation, the basic equation is unchanged:

$$\rho_1 v_1 = \rho_2 v_2 .$$

For the momentum conservation, we no longer have the pressure, but we do have a magnetic force F:

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[1] The idea is that the flow of two plasmas into each other generates instabilities that tap into the supply of kinetic energy. The electrons and ions then scatter off of the structures in the electric and magnetic fields generated by these instabilities, thereby randomizing their velocities. This process generally does not lead to complete thermalization, a subject of active research.

$$F + \frac{\partial}{\partial z}(\rho v^2) = 0.$$

We do recall however that the magnetic force is determined by the current density and magnetic field,

$$J_y = \frac{c}{4\pi}\frac{\partial B}{\partial z}, \;\; F = -\frac{1}{4\pi} B \frac{\partial B}{\partial z}.$$

(We note that since B is discontinuous at the shock, there is a δ-function in the current density along the shock.) Therefore, the momentum conservation equation tells us that $\rho v^2 + B^2/8\pi$ is conserved across the shock:

$$\rho_1 v_1^2 + \frac{B_1^2}{8\pi} = \rho_2 v_2^2 + \frac{B_2^2}{8\pi}.$$

One further equation is required in order to solve for ρ_2, v_2, and B_2. This is the equation for flux freezing: since magnetic flux through a region is conserved, if the gas is compressed by some factor in the z-direction then the magnetic field must be increased by the same factor. Thus:

$$\frac{B_2}{B_1} = \frac{\rho_2}{\rho_1} = \frac{v_1}{v_2}.$$

We may now replace all terms in the momentum equations with v_2:

$$\rho_1 v_1^2 + \frac{B_1^2}{8\pi} = \rho_1 v_1 v_2 + \frac{B_1^2}{8\pi}\frac{v_1^2}{v_2^2}.$$

If we graph the right-hand side, we see that it is a function that decreases initially, has a minimum at

$$v_{2,\min} = \left(\frac{B_1^2 v_1}{4\pi\rho_1}\right)^{1/3} = v_{\mathrm{A},1}^{2/3} v_1,$$

and then increases. We thus see that it will have at most two solutions for v_2. In fact one solution is $v_2 = v_1$. The other solution is at $v_2 < v_1$ (compressing) if $v_1 > v_{\mathrm{A},1}$ and at $v_2 > v_1$ (unphysical) otherwise. Thus a shock only propagates at velocities exceeding the Alfvén velocity (the magnetic analogue of the sound speed).

It is convenient to define the **Alfvén Mach number** $M_\mathrm{A} = v/v_\mathrm{A}$. Then if we write the ratio of velocities $r = v_2/v_1$, the velocity equation is written as

$$M_{\mathrm{A},1}^2 + \frac{1}{2} = M_{\mathrm{A},1}^2 r + \frac{1}{2r^2}.$$

The ratio r then has the solution:

$$r = \frac{1 + \sqrt{1 + 8M_{A,1}^2}}{4M_{A,1}^2}.$$

In the limit of very fast shocks, unlike the case of thermal support where $r \rightarrow 1/4$, magnetically supported shocks can achieve arbitrarily small compression ratio. This is a consequence of the fact that we have allowed the shock's energy to be radiated away.

D. C-SHOCKS

The above types of shocks contain actual discontinuities where the hydrodynamic equations break down (flow variables change over the scale of a mean free path, or faster in the case of shocks in plasmas) and are known as **J-shocks**. At low velocities in magnetized low-ionization gas, one can have another type of shock known as a **C-shock**. The C-shock provides an explanation for how H_2 observed in pre-main-sequence star outflows can survive shocks with velocities of several tens of km/s without being collisionally dissociated.

The C-shock relies on the fact that in a plasma, the electrons and ions are tied to the magnetic field lines (they spiral around them) whereas the neutrals (atoms and molecules) are oblivious to the field. On scales large compared to the mean free path, neutrals are carried with the magnetic field by collisions (or charge exchange) with the ions. On scales smaller than the mean free path, however, the neutrals simply travel on straight lines at constant velocity while the ions and electrons (tied to the field on much smaller scales, the cyclotron radii) form a "fluid." The latter is of extremely low density in weakly ionized gas, and hence it has a very large Alfvén velocity.

What happens to a shock propagating at low velocity in this medium? From a perspective of the bulk hydrodynamics, the gas can cool and so it is probably describable by the equations of §IVC. However, if one actually looks at the shock structure, one would expect the shock to occur in the ions first, as they actually form a fluid. However, the ions cannot shock if the shock velocity is less than the Alfvén velocity of the ions (i.e. using only the ion rather than the total density in the denominator). Therefore, the ions carry a **precursor** wave ahead of the shock. Behind the shock, the ions are pushed upstream by the magnetic field gradient (remember, the ion-electron plasma carries the current that supports the field!); they then collisionally transfer their momentum to the neutrals, gradually decelerating them until the velocity v_2 is reached. The gradual deceleration ensures that the neutrals are never heated to temperatures of the order of $\sim\mu v_1^2/k$.