Pulsars and Radio Wave Propagation: Probes of the ISM

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1. Introduction

We have already discussed emission spectra and the dust absorption law as probes of the ISM. It is now time for us to examine another very useful probe: **pulsars** (rotating neutron stars that emit electromagnetic radiation whose intensity varies with the rotation period of the neutron star). We will not discuss the pulsing mechanism here (which is not fully understood; theories will be discussed in Ay 125), but rather what can be learned about the ISM by studying the pulses. We will focus on the radio (even though pulsars appear at other wavelengths) because radio waves are most affected by propagation through a plasma.

References:

- Cordes & Lazio (2002), astro-ph/0207156, provides a model for the electron density spectrum described here.
- Lazio et al (2004), astro-ph/0410109, discusses the use of scattering to probe extremely small scales.

2. Dispersion Measures

Our first problem is the effect on pulses of propagation through an unmagnetized plasma.

D. RADIO WAVE PROPAGATION IN A PLASMA

We consider a plane electromagnetic wave in a stationary cold plasma, and assume the usual ~exp i($\mathbf{k}\cdot\mathbf{x}-\omega t$) space and time dependence. The wave carries an electric field, which causes the electrons to be displaced by some amount $\mathbf{s}_e$ in accordance with:

$$\ddot{\mathbf{s}}_e = -\frac{e\mathbf{E}}{m_e}.$$

(A similar result applies to the protons.) This implies that the plasma carries a current density:

$$\mathbf{J} = -ne\dot{\mathbf{s}}_e + ne\dot{\mathbf{s}}_p = -\frac{ne^2\mathbf{E}}{i\omega m_e} - \frac{ne^2\mathbf{E}}{i\omega m_p} \approx -\frac{ne^2\mathbf{E}}{i\omega m_e},$$

where n is the electron density.

Using this current density, we may now return to Maxwell's equations, which give:

$$\begin{aligned}
\nabla\times\mathbf{E} &= -\frac{1}{c}\dot{\mathbf{B}} && \rightarrow && \mathbf{k}\times\mathbf{E} = \frac{\omega}{c}\mathbf{B}\\
\nabla\times\mathbf{B} &= \frac{1}{c}\dot{\mathbf{E}} + \frac{4\pi}{c}\mathbf{J} && \rightarrow && \mathbf{k}\times\mathbf{B} = -\frac{\omega}{c}\mathbf{E} - \frac{4\pi i}{c}\mathbf{J}\\
\nabla\cdot\dot{\mathbf{E}} &= 4\pi\dot{\rho}_q = -4\pi\nabla\cdot\mathbf{J} && \rightarrow && -i\omega\mathbf{k}\cdot\mathbf{E} = -4\pi\mathbf{k}\cdot\mathbf{J}.
\end{aligned}$$

The last equation is the nontrivial one, and leads to:

$$-i\omega\mathbf{k}\cdot\mathbf{E} = \frac{4\pi ne^2}{i\omega m_e}\mathbf{k}\cdot\mathbf{E}.$$

This equation has two possible solutions. Either **k** is perpendicular to **E**, or

$$\omega^2 = \omega_p^2 \equiv \frac{4\pi ne^2}{m_e}.$$

The frequency ω_p is called the **plasma frequency**. It corresponds to the frequency of longitudinal oscillations of the plasma, in which electrons are compressed into regions of negative charge, then repel each other and bounce back, leaving a positive charge, and so on. Since the frequency does not depend on **k**, the group velocity is zero and the plasma oscillation mode does not propagate. We will therefore not consider it further.

Our attention next turns to the transverse modes, where **k** is perpendicular to **E**. These correspond to electromagnetic waves that can propagate to the observer. In this case, the first two of Maxwell's equations give:

$$\frac{c}{\omega}\mathbf{k}\times(\mathbf{k}\times\mathbf{E}) = -\frac{\omega}{c}\mathbf{E} - \frac{4\pi i}{c}\mathbf{J} = -\frac{\omega}{c}\mathbf{E} - \frac{4\pi n e^2}{\omega m_e c}\mathbf{E}.$$

The left-hand side reduces to $-ck^2\mathbf{E}/\omega$. This reduces the dispersion relation to:

$$\omega^2 = \omega_p^2 + c^2k^2.$$

That is, electromagnetic waves have a minimum possible frequency ω_p. At lower frequencies, the plasma acts as a conductor and shields incident radiation. The value of the plasma frequency is:

$$\nu_p = \frac{\omega_p}{2\pi} = 9n^{1/2}\ \text{kHz},$$

where n is in cm^{-3}. This is very low frequency compared to most ISM observations. Indeed, the Earth's ionosphere has $n \sim 10^6\ \text{cm}^{-3}$ and so frequencies below 10 MHz are unobservable from the ground.

The group velocity of a photon at frequencies $>\omega_p$ is given by:

$$v_g = \frac{d\omega}{dk} = \frac{c^2k}{\sqrt{\omega_p^2 + c^2k^2}} = c\sqrt{1 - \frac{\omega_p^2}{\omega^2}}.$$

It is of course good news that this is $<c$, and that it converges to c at the highest frequencies.

B. PULSE ARRIVAL TIMES

We now suppose that a pulsar lies a distance L away from us. Each pulse consists of a broad range of frequencies, and the highest frequencies will reach Earth first. The time required to reach us is:

$$t = \frac{L}{v_g} = \frac{L}{c}\left(1 - \frac{\omega_p^2}{\omega^2}\right)^{-1/2} \approx \frac{L}{c}\left(1 + \frac{\omega_p^2}{2\omega^2}\right) \approx \frac{L}{c} + \frac{2\pi e^2 nL}{m_e c\omega^2}.$$

The first term is an absolute offset and hence is not measurable. However, the second term is measurable: we may compare the pulse arrival times at many different frequencies and determine the coefficient nL. If the ISM has variable density, then in fact the second term contains $\int n\, dL$ instead of nL.

By this method, we may compute the **dispersion measure** (DM) to the pulsar:

$$\mathrm{DM} = \int_{\mathrm{source}}^{\mathrm{observer}} n_e \, dL,$$

where we have introduced the subscript e to remind us that it is only the free electron density that is being probed (neutral atoms produce no dispersion).

3. Rotation Measures

We now move on to the second propagation effect in a plasma: **Faraday rotation**, or the rotation of the plane of polarized light as it propagates through a magnetized medium.

D. RADIO WAVE PROPAGATION IN A MAGNETIZED PLASMA

We now suppose that the aforementioned radio wave is propagating through a plasma that contains a background magnetic field $\mathbf{B}_0$. In this case, Maxwell's equation:

$$\frac{c}{\omega}\mathbf{k}\times(\mathbf{k}\times\mathbf{E}) = -\frac{\omega}{c}\mathbf{E} - \frac{4\pi i}{c}\mathbf{J}$$

remains valid, but the relation between **J** and **E** is changed because the background field alters the motion of charged particles. In particular, the equation of motion for the electrons becomes:

$$\ddot{\mathbf{s}}_e = -\frac{e}{m_e}\mathbf{E} - \frac{e}{m_e c}\dot{\mathbf{s}}_e \times \mathbf{B}_0.$$

Replacing $\mathbf{s}_e$ with the current density $\mathbf{J} = -ne\mathbf{s}_e$, we find:

$$\frac{i\omega}{ne}\mathbf{J} = -\frac{e}{m_e}\mathbf{E} + \frac{1}{nm_e c}\mathbf{J}\times\mathbf{B}_0.$$

A general analysis of the solutions to this problem is quite complicated. However, if we work in the limit of $\omega \gg \omega_p$ and assume $\mathbf{B}_0$ to be small (so that $\omega \gg \omega_c$, where

$$\omega_c = \frac{eB_0}{m_e c}$$

is the **cyclotron frequency**). In this case, we may treat the second term in this equation as a perturbation, and write that:

$$\mathbf{J} = -\frac{\omega_p^2}{4\pi i\omega}\mathbf{E} + \frac{\omega_c}{i\omega}\mathbf{J}\times\hat{\mathbf{B}}_0 \approx -\frac{\omega_p^2}{4\pi i\omega}\mathbf{E} + \frac{\omega_p^2\omega_c}{4\pi\omega^2}\mathbf{E}\times\hat{\mathbf{B}}_0.$$

Plugging this into Maxwell's equation, we find:

$$c^2\mathbf{k}\times(\mathbf{k}\times\mathbf{E}) \approx -\omega^2\mathbf{E} + \omega_p^2\mathbf{E} - \frac{i\omega_p^2\omega_c}{\omega}\mathbf{E}\times\hat{\mathbf{B}}_0.$$

If we now fix **k** and treat ω_c as a perturbation, then there is a change $\Delta\omega$ in the frequency and a change $\Delta\mathbf{E}$ in the electric field eigenvector due to the background magnetic field. To first order in the perturbation ω_c:

$$c^2\mathbf{k}\times(\mathbf{k}\times\Delta\mathbf{E}) \approx -2\omega\Delta\omega\mathbf{E} - (\omega^2-\omega_p^2)\Delta\mathbf{E} - \frac{i\omega_p^2\omega_c}{\omega}\mathbf{E}\times\hat{\mathbf{B}}_0.$$

We focus on the transverse (perpendicular to **k**) part of this equation. In this case, the left-hand side is $-c^2k^2\Delta\mathbf{E}_\perp$, and it cancels the $\Delta\mathbf{E}_\perp$ term on the right-hand side. This leaves us with:

$$0 \approx -2\omega\Delta\omega\mathbf{E}_\perp - \frac{i\omega_p^2\omega_c}{\omega}\left(\mathbf{E}\times\hat{\mathbf{B}}_0\right)_\perp.$$

This is a 2×2 eigenvalue equation for $\Delta\omega$. Taking **k** to be in the z-direction, and $\mathbf{B}_0$ to be in the xz-plane, with angle θ to the z-axis, we find:

$$\Delta\omega\begin{pmatrix}E_x\\E_y\end{pmatrix} = -\frac{i\omega_p^2\omega_c\cos\theta}{2\omega^2}\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}\begin{pmatrix}E_x\\E_y\end{pmatrix}.$$

Note that the x-component of the background magnetic field does not enter into the equations to first order. (It does enter into $\Delta\mathbf{E}$, which requires us to use the z-component of Maxwell's equations, but we won't investigate that here.) The conclusion is that the eigenmodes are the left and right circularly polarized waves, i.e.

$$E_y = \pm iE_x,$$

with eigenvalues:

$$\Delta\omega = \pm\frac{\omega_p^2\omega_c\cos\theta}{2\omega^2}.$$

Thus, we conclude that the effect of a magnetic field is that *the left- and right-circular polarizations have slightly different frequencies*.

B. ROTATION OF THE POLARIZATION ANGLE

A linearly polarized electromagnetic wave can be thought of as the superposition of two circularly polarized waves of equal intensity. Namely, if we write the polarization vectors of the circularly polarized waves as $(1,\pm i)/\sqrt{2}$, then a linearly polarized wave at angle χ can be written as:

$$(\cos\chi,\sin\chi) = \frac{1}{\sqrt{2}}\left[e^{-i\chi}\frac{(1,i)}{\sqrt{2}} + e^{i\chi}\frac{(1,-i)}{\sqrt{2}}\right].$$

If this wave propagates through the ISM for a time t, then each component picks up an additional phase $e^{-i\omega t}$. The overall phase does not affect the polarization angle, but the additional contribution $\Delta\omega$ from the magnetic field does. Thus, the new polarization vector is:

$$\frac{1}{\sqrt{2}}\left[e^{-i(\chi+\Delta\chi)}\frac{(1,i)}{\sqrt{2}} + e^{i(\chi+\Delta\chi)}\frac{(1,-i)}{\sqrt{2}}\right],$$

where the angle by which the radiation has rotated is:

$$\Delta\chi = \frac{\omega_p^2\omega_c t\cos\theta}{2\omega^2}.$$

Thus we see that the net effect of the magnetic field is to rotate the plane of polarized light. Setting $t=L/c$, the angle can be written as:

$$\Delta\chi = \frac{2\pi e^3 nBL\cos\theta}{m_e^2 c^2\omega^2}.$$

It is common to write this instead as a function of the wavelength of the radiation,

$$\Delta\chi = \frac{e^3 nBL\cos\theta}{2\pi m_e^2 c^4}\lambda^2.$$

The amount of rotation is proportional to nBL, or more generally to the **rotation measure**:

$$\mathrm{RM} = \frac{e^3}{2\pi m_e^2 c^4}\int_{\mathrm{source}}^{\mathrm{observer}} n_e B\cos\theta\, dL.$$

As a rule of thumb, a density of $n_e = 1$ cm^{-3} over a distance of $L = 1$ pc with a magnetic field of $B = 1$ μG results in a rotation measure of 0.81 radians/m^2. (The mixed units are conventional.)

The rotation measure is observable if we have measurements of the pulsar's linear polarization angle at many wavelengths. Of course, one could be concerned about intrinsic variation of the polarization angle, but if one can observe the rotation through several cycles (as is common at low frequencies) then it is unlikely to be intrinsic.

Rotation measures require only that the source be polarized – it need not be pulsed – and so they can be determined on extragalactic radio sources as well. Of course, these may be internally Faraday rotated. They may even contain multiple components with different $n_e BL\cos\theta$, in which case the polarization angle leads to complicated functions of frequency. One often uses $d\chi/d(\lambda^2)$ as the *definition* of the rotation measure, and uses the term "Faraday depth" to denote the above integral.

4. Scintillation

We have thus far treated the ISM as having bulk properties. However, we know that the bulk velocity flows are likely dissipated on small scales as turbulence. In our own atmosphere, turbulence causes stars to twinkle. Therefore it makes sense to ask what effects interstellar turbulence would have on observations of pulsars. We will see that the pulsars can be made to twinkle as well. Before we do this, however, we need to consider the theory of turbulence.

Scintillation is generally a very complicated subject, and we will restrict our attention to order of magnitude computations.

A. TURBULENCE

Intuitively, we think of turbulent fluids (e.g. water being stirred in a cup) as having a complicated velocity field $\mathbf{v}(\mathbf{x},t)$. At any given time, the velocity field is smooth in the sense that two points close to each other in space have smaller velocity differences than points farther away. One may think of the velocity field as thus composed of eddies on various scales. Large scale eddies, which typically have large velocities, are generated by the stirring motions (in the ISM, these may be supernova explosions, or instabilities associated with Galactic rotation). These interact with each other via the convection term ($\mathbf{v}\cdot\nabla\mathbf{v}$) to transfer the energy to smaller scale eddies. Eventually, at the smallest scales, viscosity dissipates the kinetic energy of the turbulence and turns it into heat. The transfer of energy from large-scale to small-scale motions, where it is ultimately dissipated, is a key feature of turbulence and is called a **cascade**.

We may analyze this model of turbulence at order-of-magnitude level under the assumption of incompressibility (ρ=constant) and considering only hydrodynamics (no magnetic fields – yet). In particular, we may define a **turbulent dissipation rate** ε, which has units of erg cm^{-3} s^{-1}. Then we imagine that eddies of size l are associated with a characteristic velocity $v(l)$. Such eddies have kinetic energy per unit volume $\sim\rho v^2$ and transfer their energy to smaller-scale eddies in a turnover time $\sim l/v$. Therefore the energy dissipation rate is given by $\varepsilon\sim\rho v^3/l$. We thus conclude that the velocity associated with eddies at scale l is

$$v(l) \sim \left(\frac{\varepsilon l}{\rho}\right)^{1/3}.$$

We thus expect that the velocity field is indeed dominated by the largest eddies, but it contains structure at smaller scales as well. The range of validity of the above equation is called the **inertial range**. The inertial range ends when the viscous dissipation term in the velocity equation, $(\eta/\rho)\nabla^2\mathbf{v}$, is similar to the convective term, $\mathbf{v}\cdot\nabla\mathbf{v}$. This occurs when

$$\frac{\eta v}{\rho l^2} \sim \frac{v^2}{l} \rightarrow \frac{\rho l v}{\eta} \sim 1.$$

We now suppose that the turbulence is stirred at the stirring or **outer scale** L, with velocity V. Then, since $v\sim l^{1/3}$ and $lv\sim l^{4/3}$, we conclude that viscous damping occurs at the **inner scale** $l_{\rm visc}$, where:

$$l_{\rm visc} = \left(\frac{\rho L V}{\eta}\right)^{-3/4} L.$$

Due to its importance, the dimensionless ratio $\rho LV/\eta$ has a special name: it is called the **Reynolds number** (symbol: Re) and it determines the extent of the inertial range. Dimensionless numbers such as the Reynolds number play a key role in most fluid mechanics problems.

It is also possible for scalar quantities such as the entropy to be mixed by turbulence. If Q=0, then the local conservation ds/dt=0 of specific entropy implies:

$$\frac{\partial s}{\partial t} = -\mathbf{v}\cdot\nabla s.$$

In the presence of thermal conduction, a diffusion term $+\chi\nabla^2 s$ may be added. In the incompressible case, and assuming no conduction (dissipation), the integral $I = \int s^2\, d^3\mathbf{x}$ is conserved, and turbulence transfers it from large to small scales. The rate of transfer of I at scale l is $\sim\Delta s(l)^2/(l/v)$, and in statistical steady-state this should be constant with scale between the outer scale and the dissipation scale. Thus, we conclude that:

$$\Delta s(l) \propto \sqrt{\frac{l}{v}} \propto l^{1/3}.$$

Thus the 1/3 power law is expected to describe not just velocity fluctuations, but also those of scalars such as entropy. (In the Earth's atmosphere, the same law is often applicable to humidity.)

We now apply the above results to the fluctuations in the electron density n_e in the ISM. We expect that the small-scale fluctuations should be described by:

$$\Delta n_e^2(l) \sim C^2 l^{2/3},$$

where C is a normalization that has units of $\mathrm{cm}^{-10/3}$.

B. PROPAGATION THROUGH A SINGLE SCREEN

We now consider the propagation of radio waves through a turbulent ISM. To make matters simple, we will consider turbulence in a localized "screen" between the source and the observer. The source (pulsar) is treated as pointlike (we will generalize this later). The distance from the source to the screen is D_1 and from the screen to the observer is D_2. We will assume here that $D_1 >> D_2$, although in practice they are likely to be similar (this results in no qualitative changes but dramatically simplifies the math). The amplitude of the radiation received at the observer is then given by the usual diffraction formula:

$$A(\mathbf{r}) \propto \int \exp\left[i\phi(\mathbf{r}') + \frac{ik(\mathbf{r}-\mathbf{r}')^2}{2D_2}\right] d^2\mathbf{r}',$$

where $\mathbf{r}$ is the 2D position of the observer (i.e. in the plane transverse to the direction of propagation), $\mathbf{r}'$ is the 2D position in the screen, and $\phi(\mathbf{r}')$ is the phase shift introduced by the screen,

$$\phi = -k \int_{\text{screen}} \left(\frac{v_{ph}}{c} - 1\right) dx_{\|} = -\frac{2\pi e^2}{m_e c^3 k} \int_{\text{screen}} n_e \, dx_{\|}.$$

Only the phase fluctuations are observable; their variance over a transverse scale $\Delta r'$ is given by

$$\Delta\phi^2(\Delta r') \sim \left(\frac{2\pi e^2}{m_e c^3 k}\right)^2 \Delta n_e^2(\Delta r') \Delta r'^2 \frac{x_{\|}}{\Delta r'} \sim \left(\frac{2\pi e^2}{m_e c^3 k}\right)^2 C^2 x_{\|} \Delta r'^{5/3}.$$

(Here we have used the fact that the contribution to the integral is $\Delta n_e \Delta r'$ from a single eddy, and if we traverse $\sim x_{\|}/\Delta r'$ independent eddies then the variances add.)

The combination $C^2 x_{||}$ is called the **scattering measure** (SM) of the screen. It has units of cm$^{-17/3}$ but traditionally they are reported as m$^{-20/3}$ kpc. We note that the phase fluctuation variance is proportional to the scattering measure SM, to the transverse scale $\Delta r'^{5/3}$, and to λ^2.

In order to study the properties of the pulses, we first consider the situation with no ISM fluctuations (ϕ=constant). Then the integrand in A is in-phase with **r** so long as $|\mathbf{r}-\mathbf{r}'|<r_{\rm Fr}$, where $r_{\rm Fr}$ is the **Fresnel radius**:

$$r_{\rm Fr} = \sqrt{\lambda D_2}\,.$$

Outside this radius, the phase of the integrand varies rapidly and the contribution to A is therefore small. The Fresnel radius can thus be thought of as the radius of the region "explored" by radio waves as they propagate toward Earth.

C. WEAK SCINTILLATION

The key parameter controlling pulsar scintillation is the variance of ϕ evaluated at the Fresnel scale, $\Delta\phi^2(r_{\rm Fr})$. If this is small, then the pulsar undergoes **weak scintillation**. In this case, the area $\sim\pi r_{\rm Fr}^2$ that contributes to the amplitude integral undergoes fractional fluctuations of the order of $\Delta\phi(r_{\rm Fr})$. Thus the intensity of the pulses varies by an amount $\sim\Delta\phi(r_{\rm Fr})$.

The timescale of the fluctuations is determined by the timescale for the turbulent electron density pattern to change. In principle, the individual turbulent eddies could turn over, but usually the changes are dominated by the relative motion of the source, screen, and observer. The line of sight from observer to source has a transverse velocity $v_\perp$ with respect to the screen. Then the intensity fluctuations have a timescale of:

$$t_s \sim \frac{r_{\rm Fr}}{v_\perp} \sim \frac{\sqrt{\lambda D_2}}{v_\perp}\,.$$

Since $r_{\rm Fr} \propto \lambda^{1/2}$, it follows that $\Delta\phi^2(r_{\rm Fr}) \propto {\rm SM} D_2^{5/6}\lambda^{17/6}$. Thus at sufficiently short wavelengths, one has weak scintillation. As one moves to longer wavelengths, one reaches a point where $\Delta\phi^2(r_{\rm Fr})$ is of order unity. Beyond that, the phase screen has significant fluctuations at scales smaller than the Fresnel radius. This is a new regime, which we consider next.

D. DIFFRACTIVE SCINTILLATION

The scintillation of a point source when $\Delta\phi^2(r_{\rm Fr})>1$ is in the regime of **diffractive scintillation**. In this case, the phase fluctuations remain of order unity down to the diffractive scale l_d,

$$\Delta\phi^2(l_d) \sim 1, \quad l_d \sim \left[\Delta\phi^2(r_{\rm Fr})\right]^{-3/5} r_{\rm Fr} \propto ({\rm SM}D_2^{5/6}\lambda^{17/6})^{-3/5}(\lambda D_2)^{1/2} = {\rm SM}^{-3/5}\lambda^{-6/5}.$$

The existence of structure at scales l_d implies that the screen can diffract radio waves through an angle:

$$\theta_d \sim \frac{\lambda}{l_d} \propto {\rm SM}^{3/5}\lambda^{11/5}.$$

The characteristic radius that dominates the scattering is[1]:

$$|\mathbf{r}'-\mathbf{r}| \sim L \sim D_2\theta_d \sim \frac{r_{\rm Fr}^2}{l_d} \sim \left[\Delta\phi^2(r_{\rm Fr})\right]^{3/5} r_{\rm Fr} \propto {\rm SM}^{3/5}D_2\lambda^{11/5}.$$

What observable consequences result from diffractive scintillation? First, the intensity fluctuations remain of order unity, since the total amplitude is a single (complex) number that results from the coherent superposition of rays that have travelled along many different paths. But there is now a rich set of effects to consider.

Angular Broadening: We have already noted that pulsar radiation is scattered into a range of angles given by

$$\theta_d \propto {\rm SM}^{3/5}\lambda^{11/5}.$$

With interferometers it is possible to measure the broadening disks.

Pulse Broadening: A pulse propagating through the ISM is broadened because some radiation came directly from the source, whereas other radiation arrived after being scattered through an angle θ_d. Therefore pulses are temporally broadened in accordance with:

$$\Delta t \sim D_2\theta_d^2 \propto {\rm SM}^{6/5}D_2\lambda^{22/5}.$$

Frequency Coherence: The intensity scintillations do not exhibit the same pattern at all frequencies because of the *k* dependence of the amplitude integral. The patterns are coherent over a range of frequencies $\Delta\nu$, where (in accordance with the uncertainty principle):

$$\Delta\nu_{\rm coh} \sim \Delta t^{-1} \propto {\rm SM}^{-6/5}D_2^{-1}\lambda^{-22/5}.$$

Observation of the pulsar with a bandwidth exceeding $\Delta\nu$ will result in the intensity fluctuations being suppressed.

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[1] This can also be obtained by the method of stationary phase.

Temporal Coherence: The timescale of the intensity fluctuations is determined by the timescale for the screen to move by the diffraction scale. This gives:

$$\Delta t_{\rm coh} \sim \frac{l_d}{v_\perp} \propto {\rm SM}^{-3/5}\lambda^{-6/5}v_\perp^{-1}.$$

E. ORDERS OF MAGNITUDE

Scattering measures to pulsars can vary by several orders of magnitude; typical values are between 10^{-4} kpc $m^{-20/3}$ (high Galactic latitude) and 10^6 kpc $m^{-20/3}$ (for lines of sight that pass through especially turbulent clumps).

The transition frequency between weak and diffractive scintillation behavior is:

$$\nu_{\rm trans} \sim 200{\rm SM}^{6/17}D_{\rm kpc}^{5/17}\ {\rm GHz}.$$

Thus most pulsar observations are in the diffractive regime.

The parameters in the diffractive scintillation regime can then be written as:

$$\begin{aligned}
&l_d \sim 900\nu_9^{6/5}{\rm SM}^{-3/5}\ {\rm km}\\
&\theta_d \sim 0.07\nu_9^{-11/5}{\rm SM}^{3/5}\ {\rm arcsec}\\
&\Delta t \sim 1.1\nu_9^{-22/5}{\rm SM}^{6/5}D_{\rm kpc}\ {\rm ms}\\
&\Delta\nu_{\rm coh} \sim 170\nu_9^{22/5}{\rm SM}^{-6/5}D_{\rm kpc}^{-1}\ {\rm Hz}\\
&\Delta t_{\rm coh} \sim 90\nu_9^{6/5}{\rm SM}^{-3/5}v_{\perp,4}^{-1}\ {\rm s}.
\end{aligned}$$

F. EXTENDED SOURCES

We have considered scintillation of pulsars. However, scintillation applies to any sufficiently small object (although of course pulse broadening times only exist for pulsars or radio transients). An extragalactic source will appear broadened by the scattering disk θ_d if its intrinsic angular size is $<\theta_d$. This applies to many AGNs, whose central engines are unresolved even on long baselines. Of course, the extended emission (e.g. synchrotron lobes) usually is at larger sizes.

A related question is whether the source will undergo intensity fluctuations. If a point source is moved by an angular distance $\sim\lambda/L$, then the relative phase of different rays that contribute to the overall propagation amplitude is shuffled. Therefore, the fluctuations de-correlate between two point sources separated by this distance. Since an extended source can be thought of as many point sources, we conclude that the intensity fluctuations are suppressed for sources larger than λ/L. We note that λ/L is tiny:

$$\frac{\lambda}{L} \sim \frac{\lambda}{D_2\theta_d} \sim 0.006\nu_9^{11/5}\mathrm{SM}^{-3/5}D_{\mathrm{kpc}}^{-1}\ \mu\mathrm{arcsec}.$$

Thus at high Galactic latitude, intensity scintillations are sensitive to the angular sizes of objects on scales of *microarcseconds*.

Larger objects will still undergo scintillation because the spectrum of fluctuations contains turbulent eddies that magnify or demagnify the entire scattering disk. These fluctuations are a result purely of geometric optics and are called **refractive scintillation**.