AY 102
HW #7
Due: Wednesday, March 9, 2010

#1. Shock heating of gas with general polytropic index. [15 points]

Consider a gas whose internal energy per unit mass, rather than being $3kT/2\mu$, is

$$e = \frac{1}{\gamma - 1}\frac{kT}{\mu},$$

where γ is a constant.

(a) Show that $h/e=\gamma$.

(b) Repeat the analysis of the nonradiative strong shock ($v_1 >> c_s$) for arbitrary γ. What is the compression factor? The post-shock temperature T_2?

(c) Evaluate T_2 for the following cases:

> [i] A stream of cold primordial gas travelling at 150 km/s collides with the disk of a galaxy.
>
> [ii] A spacecraft re-enters the Earth's atmosphere (diatomic; N_2 + O_2; $\gamma=9/7$ if the gas is hot enough to excite vibrational modes) from orbit. It produces a shock that travels at the spacecraft's velocity (7.5 km/s).[1]

#2. Modification to Supernova Remnant Evolution. [15 points]

Suppose that instead of exploding in a uniform medium, a supernova explodes in a medium that is spherically symmetric with density profile

$$\rho(r) \propto r^{-s}.$$

For example, $s=2$ would correspond to a steady pre-SN stellar wind.

Compute the power-law exponents for how the radius of the blast wave R, the shock velocity v_1, and the post-shock temperature T_2 vary as a function of time t for each of the major supernova remnant phases discussed in class (free expansion; energy-conserving/Sedov-Taylor; cooling of shocked ISM gas; and cooling of the hot central bubble).

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[1] The answer you get here will be an overestimate because some of the energy is taken up by dissociation of O_2.

#3. Faraday Rotation. [10 points]

(a) AGN jets are sometimes theorized to contain a pair plasma (i.e. equal numbers of e^+ and e^-). Show that (to linear order in the magnetic field) such a plasma does not lead to Faraday rotation.

(b) The Earth's daytime ionosphere has a typical free electron density of $\sim 10^6$ cm^{-3} and a thickness of ~100 km. The Earth's magnetic field is ~0.3 G. Estimate the critical frequency above which radio polarization is significantly rotated (≥1 radian) by the ionosphere.