AY 102
HW #5
Due: Wednesday, February 23, 2011

#1. Neutral ISM phases. [13 points]

(a) Using the formulae in the notes, write down an expression for the total cooling (in erg s^{-1} per H nucleus) of neutral gas as a function of temperature T (in K) and pressure P/k (in units of K cm^{-3}, with Boltzmann's constant divided out). Include the cooling in the fine structure lines of C II and O I, the optical forbidden doublet [O I] 6300, 6364 Å, and H I Lyα. Assume that we are below the critical density for all of these lines; in which case is this most restrictive?

(b) Plot this on a log-log scale as a function of temperature and compare to the typical photoelectric heating rate. What is the range of pressures over which you predict that two phases are allowed?

#2. Energy levels of molecules. [12 points]

In dense environments, it is a common occurrence for molecules to reach a thermal distribution of rotational energy levels.

(a) At T = 20 K – a typical temperature deep inside a molecular cloud – compute the probability distribution for J in the $^{12}C^{16}O$ molecule.

(b) Draw qualitatively the equilibrium ratio of ortho-H_2:para-H_2 as a function of T. At what temperature is the ratio 1:1?

#3. Deuterium. [15 points]

Deuterium is rare in the ISM – the primordial D/H ratio was $\sim 2\times 10^{-5}$ and D has only been destroyed by stars. Nevertheless, deuterated molecules have been detected. Assuming that the H−X bond (where X is a heavy atom or radical) oscillates at angular frequency ω, compute:

(a) The frequency of oscillation of the D−X bond.
(b) The difference in ground state energies of the hydrogenated versus deuterated molecule.
(c) The equilibrium constant of the reaction:[1]

$$HX + D^+ \rightleftharpoons DX + H^+.$$

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[1] For the purposes of this simplified problem (but not necessarily the purposes of real life) you may ignore any possible H or D atoms in the radical X and the consequent wave function (anti)symmetrization requirements.

(d) The value of this equilibrium constant for a typical H−X bond (which emits radiation at ~3000 waves/cm) and a temperature of T = 20 K.

The strong preference for deuterating molecules can lead to some amazing molecules, including *fully* deuterated ammonia, ND_3! (Lis et al, ApJ 571, L55, 2002)