AY 102
HW #4
Due: Wednesday, February 9, 2010

#1. Thermal Fluctuations in Large Grains. [20 points]

Consider a dust grain with radius a greater than the critical radius discussed in class. Assume that the grain emits in the FIR with total luminosity $L \sim T^{4+\beta}$. Let the equilibrium dust temperature be $T_{d,eq}$ and its equilibrium luminosity (i.e. the rate of absorption of energy from starlight) be L_{eq}. Unless otherwise specified, later parts of the problem may be answered in terms of the results from earlier parts.

(a) Use the Debye model to calculate the heat capacity C of the grain at the equilibrium temperature in terms of its sound speed c_s and the aforementioned parameters.

(b) If the temperature is perturbed from its equilibrium value by an amount ΔT_d, calculate the time constant τ_T for the grain to restore its equilibrium temperature. You may assume $|\Delta T_d| << T_d$.

(c) Suppose that the grain actually receives photons stochastically, i.e. it receives energy in packets of energy E_{UV}, at a mean rate $\Gamma_{UV} = L_{eq}/E_{UV}$. Derive a formula for the value of the grain temperature perturbation $\Delta T_d(t)$ at time t in terms of the arrival times of all previous photons. [*Hint*: Remember that you are computing $\Delta T_d(t)$, and hence in addition to considering the discrete photon absorptions you must subtract off the mean energy input.]

(d) Evaluate the variance of your result from (c). What is $\sigma(T_d)/T_{d,eq}$? Express this in terms of β, E_{UV}, and the equilibrium energy content of the grain U_{eq}.

(e) Numerically evaluate (d), to an order of magnitude, for a = 0.2 μm silicate grains in the diffuse ISM.

#2. Interplanetary Dust. [14 points]

Consider a silicate dust grain of radius a = 2 μm in the solar system. Make any approximations necessary to complete the problem.

(a) If the grain is 1 AU from the Sun, what is its equilibrium temperature?

(b) What is the radiation-induced force on the grain? How does it compare to the gravitational attraction by the Sun? Can the grain be bound to the Solar System?

#3. Absorption of the CMB by Dust. [6 points]

In class we described the intrinsic spectrum of the dust, and approximated its low-frequency behavior by a modified blackbody, $j_\nu \propto \nu^\beta B_\nu(T_d)$. This implies an opacity at low frequencies, and hence in principle the dust "emission" spectrum we measure should be corrected by correcting for the absorption of the cosmic microwave background. The CMB is a uniform blackbody of temperature 2.7 K that is behind all of the dust. Compute, for T_d = 18 K, the fractional correction due to CMB absorption at 90, 140, and 220 GHz.