AY 102
HW #3
Due: Wednesday, February 2, 2011

#1. Cascade Matrices. [13 points]

A hydrogen atom recombines to the 5d level. Assume that Case A applies and that the density is low enough to neglect collisions. For this problem, you may look up the decay rates for hydrogen on the course website (or anywhere else).

(a) What are the possible levels that the atom could pass through on its way to the ground state?

(b) Compute the cascade matrices $C(5\text{d},nl)$ for these levels.

(c) What is the probability for the hydrogen atom to finally decay to the ground state by two-photon emission versus Lyman-α?

#2. Absorption of Lyman-α Radiation. [13 points]

Suppose that we observe a nearby quasar in the ultraviolet. Its intrinsic spectrum (i.e. before considering absorption by atomic hydrogen in the Milky Way) is $S_\nu = S_0$ = constant, and we observe it through a column density of atomic gas $N_{\rm HI} = 8 \times 10^{19}\ \text{cm}^{-2}$ (typical for moderate Galactic latitude) in our own Galaxy. You may assume the gas to have a turbulent velocity dispersion σ_v = 20 km/s and negligible thermal velocities.

(a) Calculate the natural and Doppler broadening of the Lyman-α line.

(b) Make a plot of the optical depth τ_ν due to absorption in the Lyman-α line and the observed spectrum S_ν in the relevant part of the spectrum.

(c) How wide is the range of frequencies $\Delta\nu$ that is more than 50% absorbed? What is $\Delta\nu/\nu_{\rm Ly\alpha}$?

#3. Resonances in the Electric Susceptibility. [14 points]

Consider a material with a number density n of oscillators. Each oscillator is taken to be an electron with charge $-e$, mass $m_{\rm e}$, and sits in a potential of spring constant $m_{\rm e}\omega_0^2$. Additionally it feels a damping force $\mathbf{F}_{\rm damp} = -m_{\rm e}\gamma\mathbf{v}$ (of unspecified origin), with $\gamma \ll \omega_0$.

(a) Derive the electric susceptibility and dielectric constant of the material as a function of frequency.

(b) Taking Im $\chi(\omega)$ from (a), use the Kramers-Kronig relations to derive Re $\chi(\omega)$. Show that this agrees with your result from (a).