AY 102
HW #2
Due: Wednesday, January 26, 2011

#1. Free-free radiation. [20 points]

This problem will walk you through the derivation of the radio luminosity due to free-free emission from a fully ionized hydrogen plasma. Classical physics and order-of-magnitude arguments may be used throughout this problem. You may also assume that the protons are stationary since their velocities are much less than those of the electrons under thermal conditions.

(a) Suppose an electron undergoes a sudden change of velocity $\Delta\mathbf{v}$. Assume that the energy radiated, in units of ergs per logarithmic range in frequency, can be calculated from the standard dipole formula by smoothing the change in velocity over a time interval ν^{-1}. What is this radiated energy? Use this to infer $dE/d\nu$ (units: erg Hz^{-1}). Over what range of frequencies is this argument valid?

(b) Suppose that an electron of velocity v is incident on a proton with impact parameter b. Compute the change in velocity during the encounter, $\Delta\mathbf{v}$. Assume in this calculation that the deflection angle of the electron is << 1 radian; what constraint does this impose on v and b?

(c) Compute the emitted energy per unit frequency $dE/d\nu$ in the collision described in part (b). For a give frequency ν and relative velocity v, give the maximum and minimum impact parameters at which the calculation is valid.

(d) Assuming all electrons have the same velocity v, and assuming a number density n for both electrons and protons, calculate the rate of emission of radio waves $4\pi j_\nu$ (in erg cm^{-3} s^{-1} Hz^{-1}). [Hint: You will need to do an integral over the impact parameter.]

(e) What is the scaling of your result with n and T?

#2. Photoionization models. [20 points]

(a) Download and install CLOUDY on your favorite computer. Generate the radial ionization and temperature structure for a spherically symmetric model nebula with a star of central temperature 5×10^4 K, an H-ionizing photon emission rate of $10^{49.7}$ s^{-1}, a hydrogen density of $n_H=10^2$ cm^{-3}, and an inner radius of $r_{in} = 10^{17}$ cm. Attach plots of $T(r)$ and the H, He, C, and O ionization stage fractions as functions of radius. Qualitatively explain their features.

(b) What does the standard Strömgren sphere calculation from class predict for the outer radius of the H II region? How does this compare with the CLOUDY results?

Technical notes:

- Make sure you read the documentation (for this assignment, the Quick Start guide will be enough – the more advanced topics will make sense later in the course).
- You will want to set the "sphere static" command to set the velocity field to zero and impose spherical symmetry.
- You will need to adjust the number of iterations to make sure your results are converged.
- The default is to stop the calculation at $T = 4000$ K. You should find that this is near the transition to the neutral H^0 zone.
- Some of CLOUDY's defaults that you will *not* want to change for this problem include the use of (roughly) solar metallicity, no dust, etc. In general, this code has *far* more capability than you will use for this class.