AY 102
HW #1
Due: Wednesday, January 19, 2011

#1. Lyman-α decay. [12 points]

(a) Look up in your quantum mechanics textbook the wave functions for the 1s and $2p_z$ orbitals of hydrogen.

(b) Calculate the matrix element $\langle 1s|\mathbf{x}|2p_z\rangle$.

(c) Use this to obtain the $2p_z \rightarrow 1s$ decay rate in hydrogen.

#2. Alternative derivation of the *J*-values for O II. [12 points]

(a) Calculate the number of electrons k in the 2p subshell in the lowest configuration of O II.

(b) There are six 2p spin-orbitals (i.e. possible values of m_l and m_s for a single electron). Explicitly write down the $\binom{6}{k}$ ways to put k electrons in these spin-orbitals and calculate the values of M_J, the projection of total angular momentum onto the z-axis, for each one. Make a histogram of the M_J values.

(c) Using the fact that each energy level with total angular momentum J has $2J+1$ quantum states with M_J ranging from $-J \ldots J$, determine using your answer to part (b) which values of J exist in the lowest configuration of O II, and how many energy levels exist for each J. Compare this to the energy level diagram from the notes.

#3. Ionization energies of helium-like ions. [16 points]

In this problem, we will investigate the ionization energies of ions with two electrons and atomic number Z. You may, for these purposes, take the orbital part of the ground-state wave function to be $\Psi_{\rm orb}(\mathbf{x}_1,\mathbf{x}_2) = \psi_{1s}(\mathbf{x}_1)\psi_{1s}(\mathbf{x}_2)$, where ψ_{1s} is the wave function of a 1-electron (hydrogen-like) ion with atomic number Z.

(a) What are the spin wave function and term symbol?

(b) Compute the approximate energy by taking the expectation value of the Hamiltonian, $\langle \Psi_{\rm orb}|H|\Psi_{\rm orb}\rangle$. You may leave your answer in terms of the integral $I = \int \psi_{1s}^*(\mathbf{x}_1)\psi_{1s}^*(\mathbf{x}_2)\frac{e^2}{r_{12}}\psi_{1s}(\mathbf{x}_1)\psi_{1s}(\mathbf{x}_2)d^3\mathbf{x}_1 d^3\mathbf{x}_2$. [*Hint*: The Hamiltonian is given in the lecture notes; all of the terms involving a single electron can be solved using the fact that ψ_{1s} is an eigenfunction of the operator $H_1 = -(\hbar^2/2m_e)\nabla_1^2 - Ze^2/r_1$.]

(c) Without any detailed calculations, explain why I is equal to some fixed positive numeric constant times Ze^2/a_0.

(d) Compute the energy difference between the 2-electron and 1-electron ions. [For this part, you may use the fact that the constant in (c) turns out to be ⅝.]

(e) Evaluate numerically the expression in (d) for the ionization energies of the sequence He … O^{6+} and compare them to tabulated values.