

Cosmic Rays: I. General Phenomenology, Energy Loss, and Electromagnetic Signatures

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1. Introduction

In class, we have primarily discussed the thermalized particles in the ISM. However, very fast-moving particles lose energy very slowly via collisions, and thus can survive for long timescales before slowing down. These particles are called **cosmic rays**. Cosmic rays play a key role in the pressure support of the ISM; they are a source of ionization and heating in regions of low UV radiation density (i.e. in dark clouds), and are responsible for several of the key emission processes in the ISM (particularly synchrotron radiation and several of the gamma ray emission processes). Most cosmic rays are protons, but electrons and heavy nuclei can also be accelerated.

In this set of lectures, we will consider the phenomenology of the cosmic ray spectrum. Then we will cover the mechanisms by which cosmic rays lose energy (some of which lead to electromagnetic radiation, which is directly visible). Then we will consider their acceleration and propagation (described last as this is the least well-understood process).

A short summary of the locally observed CR population is the Particle Data Group review:

<http://pdg.lbl.gov/2009/reviews/rpp2009-rev-cosmic-rays.pdf>

Some references on energy loss mechanisms:

- Rybicki & Lightman, *Radiative Processes in Astrophysics*: The standard guide to synchrotron and inverse Compton radiation, and their application to a number of environments (including the ISM but also AGNs and high-energy astrophysics).

- Leo, *Techniques for Nuclear and Particle Physics Experiments*: Chapter 2 has a number of useful formulas for energy loss mechanisms of particles in laboratory situations. It's a good reference if you are interested in working on cosmic ray experiments.

Much more detail can be found on the Galprop webpage and references therein:

http://galprop.stanford.edu/web_galprop/galprop_home.html

2. General Phenomenology

The cosmic ray population is one of the few ISM components that we can directly measure. This is because some CRs can propagate unimpeded into the Solar System. Nevertheless, what we observe is not unaffected by our local environment: the radius of curvature r_L of a CR track in a magnetic field is

$$r_L = \frac{R}{B}, \quad R = \frac{cp}{Ze},$$

where R is the **magnetic rigidity** of a CR with momentum p and charge Ze . Particles with p/Z below a few GeV/ c are efficiently deflected by the Earth's magnetic field; this effect is reduced for measurements in interplanetary space, but CRs still need to propagate upstream through the solar wind, and the flux of CRs below a few GeV/ c is still reduced relative to its interstellar value (and in fact varies with the 11-year solar cycle: more cosmic rays can propagate into the solar system at solar minimum).

The cosmic rays are composed mainly of nuclei, with electrons, positrons, and antiprotons as minor constituents. The intensity of cosmic ray nuclei at energy above a few GeV per nucleon (and up to ~ 100 TeV/nucleon) is:

$$I_N(E)dE \approx 1.8 \times 10^4 E_{\text{GeV}}^{-2.7} \frac{\text{nucleons}}{\text{m}^2 \text{ s sr GeV}},$$

where E is the energy *per nucleon* of the cosmic ray.

By nucleon count at ~ 10 GeV/nucleon, the CRs consist of $\sim 76\%$ H, $\sim 15\%$ He, and $\sim 4\%$ C/N/O nuclei. The remaining $\sim 4\%$ are a mixture of other nuclei extending up to Ni. Interestingly, $\sim 0.5\%$ of the CR nucleons are in Li/Be/B isotopes, which are extremely rare as they are not produced in stars. These are indicative of production of these isotopes by nuclear reactions involving cosmic rays.

Some radioactive isotopes are observed in the CR population, e.g. ^{10}Be ; since these are also produced by nuclear reactions involving cosmic rays, the ratio of their abundance to that of stable isotopes serves as a clock that enable radioactive dating of CRs. Such methods suggest an age for the CR population of ~ 15 Myr, so it is clear that our Galaxy's CR population is continuously being generated, and that older CRs have been "lost" (due to either cooling or escape from the Galaxy).

At a few GeV, the local CR electron/proton ratio is $\sim 1\%$, but this decreases slowly with energy above 10 GeV. This extremely small ratio must (and can) be explained by theories of cosmic ray acceleration and transport; the basic concepts are that due to their smaller initial momentum electrons are harder to accelerate than heavy particles (protons, nuclei), and that at the highest energies they suffer far more energy loss in the ISM.

Antimatter is also present in the CRs. Positrons are also present; the PAMELA satellite experiment measuring an e^+/e^- ratio rising from $\sim 5\%$ at <10 GeV to $\sim 10\%$ at ~ 100 GeV. Antiprotons have an abundance relative to protons of $\sim 2 \times 10^{-4}$ at 10—20 GeV, with a fall-off at lower energies; this is expected if the main mode of production of antiprotons is due to collisions of very high energy cosmic rays with target nuclei in the ISM. There is as yet not a universally accepted explanation for the rise in the positron spectrum observed by PAMELA.

3. Energy Loss Mechanisms

We will now consider the energy loss mechanisms for cosmic rays. These fall into several categories: losses due to electromagnetic interactions of charged particles with the medium in which they propagate; radiative processes (for electrons); and nuclear collisions.

A. ELECTROMAGNETIC COLLISIONS

We consider the motion of a cosmic ray with charge Ze through a plasma with density n . Our objective is to determine the rate at which energy is lost. For simplicity, we will do a quasiclassical nonrelativistic calculation, which reproduces the key features of the quantum mechanical calculation.

Consider first an electron at rest, with the cosmic ray passing it at impact parameter b and velocity v . The electron feels an electric field $E \sim Ze^2\gamma/b^2$ for a duration of time $\Delta t \sim 2b/v\gamma$, where $\gamma = (1 - v^2/c^2)^{-1/2}$. It thus acquires a velocity

$$v_e \sim \frac{eE\Delta t}{m_e} = 2 \frac{Ze^2}{m_e b v}.$$

The kinetic energy imparted to the electron is then:

$$\Delta E_e \sim \frac{1}{2} m_e v_e^2 \sim 2 \frac{Z^2 e^4}{m_e b^2 v^2}.$$

Our objective is to determine the rate of loss of energy. This can be obtained by summing the individual energy loss for each interaction:

$$\frac{dE}{dx} = \int 2\pi b \Delta E_e n db \sim 4\pi \frac{Z^2 e^4 n}{m_e v^2} \ln \Lambda,$$

where $\Lambda = b_{\max}/b_{\min}$ is the range of impact parameters over which the above formulas are valid. Typically, $b_{\min}$ is determined by the impact parameter at which there is a “hard” collision (deflection angle of order unity); and $b_{\max}$ is the impact parameter at which either (i) the orbit of the electron in an atom has a timescale $< \Delta t$ (for bound electrons), or (ii) the plasma interaction screens the electric field of the cosmic ray (for free electrons). We will not dwell on the computation of $\ln \Lambda$ here since it is almost always between a few and a few tens.

We may compute the range of a particle by dividing its energy by the energy loss rate. Recalling that for nonrelativistic particles $E \sim Mv^2/2$ (where M is the mass of the cosmic ray),

$$\Delta x \sim \frac{E}{dE/dx} \sim \frac{m_e v^2 E}{4\pi Z^2 e^4 n \ln \Lambda} \sim \frac{m_e E^2}{2\pi M Z^2 e^4 n \ln \Lambda}.$$

The timescale for the cosmic ray to stop is

$$\Delta t \sim \frac{\Delta x}{v} \sim \frac{m_e E^{3/2}}{2\pi M^{1/2} Z^2 e^4 n \ln \Lambda}.$$

Thus we see the critical feature that allows cosmic rays to propagate: as their energy increases, collisional energy loss becomes *less* important. For e.g. a 100 MeV proton in a density of 1 cm^{-3} , we find a range of $\Delta x \sim 1 \text{ Mpc}$ and a stopping timescale of $\Delta t \sim 10 \text{ Myr}$. Thus we can see that electromagnetic interactions are very inefficient at stopping the fastest particles.

For *relativistic* particles it turns out that the energy loss rate stops decreasing, i.e. when $E > Mc^2$, dE/dx becomes flat (and may actually rise slightly due to changes in $\ln \Lambda$). In this case, Δx and (measured in the lab frame) Δt then increase only linearly with E instead of quadratically. However, in this case there are usually more important mechanisms of energy loss: nuclear interactions in the case of CR protons/nuclei, and radiative interactions in the case of electrons/positrons.

B. SYNCHROTRON RADIATION

Relativistic electrons and positrons lose energy primarily by radiative processes. There are 3 categories: **synchrotron radiation** due to the interstellar magnetic field; **inverse-Compton radiation**, in which ambient radiation (CMB, starlight, IR from dust) is scattered to a higher frequency by the fast-moving particle; and **bremsstrahlung** (emitted by the electron or positron in collisions with particles in the ISM). These mechanisms all depend on inverse powers of M and hence are negligible for protons.

Synchrotron radiation arises from the interaction of a relativistic electron with the interstellar magnetic field. We consider an electron at Lorentz factor $\gamma \gg 1$ moving through the ISM. In its own rest frame, it sees an electric field:

$$\mathbf{E}' = \frac{\gamma \mathbf{v} \times \mathbf{B}}{c},$$

where the primes denote quantities measured in the electron frame. The electron then accelerates at:

$$\mathbf{a}' = -\frac{e\gamma}{m_e c} \mathbf{v} \times \mathbf{B},$$

and consequently radiates power

$$P' = \frac{2e^2 a'^2}{3c^5} = \frac{2e^4 \gamma^2 v^2 B^2 \sin^2 \theta}{3m_e^2 c^5} \approx \frac{2e^4 \gamma^2 B^2 \sin^2 \theta}{3m_e^2 c^3}.$$

This power is measured in the electron frame; however, we know that in the lab frame the power will be the same¹ (assuming an equal amount of radiation is emitted forward and backward in the electron rest frame), so the total power emitted by the electron is

$$P = \frac{2e^4 \gamma^2 B^2 \sin^2 \theta}{3m_e^2 c^3}.$$

The power emitted is thus proportional to γ^2 . The timescale to stop a particle is roughly given by:

$$\tau_{\text{sync}} = \frac{E}{P} = \frac{m_e c^2 \gamma}{2e^4 \gamma^2 B^2 / 3m_e^2 c^3} = \frac{3m_e^3 c^5}{2e^4 B^2 \gamma} = 8 \times 10^{12} B_{\mu\text{G}}^{-2} \gamma^{-1} \text{ yr}.$$

The slow-moving electrons are thus more substantially affected by electromagnetic collisions, whereas the fast-moving electrons are slowed primarily by synchrotron radiation. In a magnetic field of $\sim 6 \mu\text{G}$, a 100 MeV electron is slowed by synchrotron losses in 10^9 years, a 10 GeV electron is slowed in 10^7 years, and a 1 TeV electron would be slowed in only 10^5 years.

We also need to know the typical frequency of synchrotron radiation. This can be understood heuristically as follows. We know that radiation from a highly relativistic source is typically emitted in a range of directions of width $\Delta\theta \sim \gamma^{-1}$

¹ The *energy* of emitted radiation is enhanced by a factor of γ in going from the moving to the lab frame; but the *time* over which it is emitted is also enhanced, so $P' = P$.

around the velocity vector, since the transverse components of the momentum of the radiation are not affected by the Lorentz transformation but the component parallel to the direction of propagation receives a $\sim\gamma$ boost. Thus we see synchrotron radiation from a fraction $\sim\gamma^{-1}$ of the electron's orbit. But the orbit of the electron has a frequency given by

$$\frac{d\mathbf{p}}{d\tau'} = -e \frac{\gamma \mathbf{v} \times \mathbf{B}}{c} = -e \frac{\mathbf{p} \times \mathbf{B}}{m_e c},$$

where τ' is *proper* time. Thus the frequency of the orbit as measured on the electron is $\sim eB/m_e c$ (we will neglect the factor $\sin \theta$ here as it is typically of order unity), so the burst of synchrotron radiation that we see has a duration in the electron frame of

$$\Delta t' \sim \frac{m_e c}{eB\gamma}.$$

The frequency of the radiation is then determined by the uncertainty principle: $\omega' \sim 1/\Delta t'$. This frequency transformed into the lab frame is enhanced by a factor of γ by the Lorentz boost, so

$$\omega \sim \frac{\gamma}{\Delta t'} \sim \frac{eB}{m_e c} \gamma^2.$$

We thus expect a typical frequency

$$\nu \sim \frac{eB}{2\pi m_e c} \gamma^2 = 3B_{\mu G} \gamma^2 \text{ Hz}.$$

This is correct, although there is actually a range of frequencies, even from a single-energy electron. But we can see that the electrons contributing to the 408 MHz map have $\gamma \sim 5000$ or energies of 2—3 GeV. The electrons that contribute to synchrotron in the microwave bands (30 GHz) have energies of order ~ 20 GeV.

Synchrotron radiation is polarized perpendicular to the magnetic field since the polarization is determined by the direction of the acceleration vector of the charge. Detailed calculations show that the fractional polarization depends on the electron spectrum, but values around $\sim 75\%$ are typical. The observed polarization may be much less because one looks through regions of different magnetic field direction, or different amounts of Faraday rotation.

C. INVERSE COMPTON RADIATION

A third mechanism of energy loss from relativistic electrons is inverse-Compton scattering. The idea here is that there is a background of soft photons with some energy density u – whether they be the optical radiation from stars, the

infrared radiation from dust, or the CMB. A relativistic electron plows through this soft radiation field, and in its rest frame, they all appear to be coming from “ahead” (and with an energy density $u\gamma^2$: there is one factor of γ associated with the increase of energy in each photon, and another factor of γ associated with length contraction). The scattering of these photons on the electron leads to a retarding force

$$F' = u\gamma^2\sigma_T,$$

where σ_T is the Thomson cross section. This implies an amount of power lost by the electron of

$$P = cF' = u\gamma^2\sigma_T c.$$

Like synchrotron, this power is proportional to γ^2 , so the ratio of inverse-Compton to synchrotron losses is independent of the energy of the electron. We may write:

$$\frac{P_{\text{IC}}}{P_{\text{sync}}} \sim \frac{u\gamma^2\sigma_T c}{e^4\gamma^2 B^2 / m_e^2 c^3} \sim \frac{u}{B^2} \sim \frac{u}{u_B},$$

where u_B is the magnetic energy density. Thus inverse-Compton losses dominate over synchrotron if the ambient radiation energy density is greater than the magnetic energy density. In the ISM we typically find $u_B \sim 10^{-12}$ erg/cm³, as compared with the energy density of the CMB (4×10^{-13} erg/cm³) and of starlight ($\text{few} \times 10^{-12}$ erg/cm³). Thus we expect similar energy losses in synchrotron and inverse-Compton.

This of course begs the question of at what frequency the inverse-Compton radiation from CR electrons could be observed. In the rest frame of the electron, the input photon’s frequency is increased by a factor of γ relative to the lab frame; and after scattering through a large angle, if we go back to the lab frame the photon’s energy is boosted by another factor of γ . (An exception to this statement occurs if the photon energy in the rest frame becomes of order $m_e c^2$, in which case the electron recoil is relativistic. This requires the use of the relativistic **Klein-Nishina** formula for the cross section rather than Thomson, which is relevant in some extreme circumstances but is not described in this class.) Thus there is a net boost of a factor of γ^2 . For electrons at ~ 20 GeV (the WMAP bands), $\gamma^2 \sim 10^9$. Thus a CMB photon with an input energy of 10^{-3} eV would be scattered up into the soft γ -rays (~ 1 MeV). In practice the emission from our Galaxy in this waveband is dominated by X-ray binaries and so the up-scattered CMB is unobservable. However, up-scattered starlight photons start at 1 eV and are thus scattered to 1 GeV; these are a significant (but not dominant) contribution to the hard γ -ray emission from our Galaxy.

D. BREMSSTRAHLUNG

A third mode of energy loss from electrons is bremsstrahlung: the burst of radiation emitted by the electron as it passes through the electric field of a particle in the ISM (electron or nucleus) and undergoes transient acceleration.

The energy loss rate due to bremsstrahlung will be calculated at order-of-magnitude level on HW#7. For here, we will just give the two key results:

- ⇒ An electron of energy $E = m_e c^2 \gamma$ loses a fraction of order unity of its energy via bremsstrahlung after traveling a distance equal to the **radiation length**,²

$$L_{\text{rad}} = \left[4 \sum_Z n_Z Z(Z+1) \alpha r_e^2 \ln \frac{2E}{m_e c^2} \right]^{-1},$$

where n_Z is the number density of nuclei with atomic number Z , α is the fine structure constant, and r_e is the classical electron radius.

- ⇒ Most of the emitted energy is in γ -rays with a fraction of order unity of the incident electron energy E .

The radiation length in an ISM composed of hydrogen at $n_H = 1 \text{ cm}^{-3}$ is 10^{27} cm and the associated bremsstrahlung cooling time is 10^9 years. It is thus significant only for the slower-moving electrons whose synchrotron/inverse-Compton cooling times are longer than this; in particular the ISM γ -ray emission in the tens of MeV band contains a large contribution from bremsstrahlung.

E. NUCLEAR COLLISIONS

The radiative processes – synchrotron, inverse Compton, and bremsstrahlung – apply to the CR electrons. But most CRs are *nuclei*, and as such are too heavy to participate in the above processes. At low energies, they may lose energy by electromagnetic collisions, but at high energies they instead collide with stationary nuclei in the ISM. For example, the geometric cross section of a proton (radius 1 fermi) is $\sim 3 \times 10^{-26} \text{ cm}^2$. Thus at a density of 1 cm^{-3} , a CR can traverse $\sim 3 \times 10^{25} \text{ cm}$ (or 10 Mpc, taking a time of order 30 Myr) before it collides with a nucleus. If the CR is a nucleus and the speed is relativistic, then the Coulomb repulsion of the nuclei is small; they will collide, and since they are strongly interacting it is likely that the collision will be inelastic. We will discuss both purely nuclear reactions here, as well as the production of new elementary particles. A key feature is the production of new types of particles in the CR population that were

² In materials that are not fully ionized, and for very fast electrons $\gamma \gg \alpha^{-1} Z^{-1/3}$, the fact that the electrons are clustered around the nuclei reduces the electric fields that cause bremsstrahlung, and hence result in an increase in L_{rad} . In such cases, L_{rad} becomes independent of energy at high E (instead of decreasing), and it is this high-energy limit that high energy experimentalists generally refer to as the “radiation length.”

not part of the initial seed population that was accelerated (the **primary cosmic rays**); these new particles are called **secondary cosmic rays**.

Nuclei: We are all familiar with fusion reactions between nuclei in stars at temperatures of 1 to a few hundred keV. At relativistic energies (hundreds of MeV or more per nucleon), reactions tend to be much more destructive: nucleons or small particles (e.g. α particles) may be expelled, and the remains of the projectile and target will carry large amounts of internal energy. They may de-excite by a combination of evaporating nucleons and γ -ray emission. The result is the conversion of heavier nuclei into smaller nuclei.

Some of the secondaries produced in this way are of minor interest. For example, the primary CRs contain many p (^1H) and α (^4He) particles, so while some are secondary the production is not especially remarkable. Similarly, neutrons are produced, but these β -decay rapidly into protons. However, some rare nuclei can be produced by CR collisions. For example, the light elements (^6Li , ^9Be , ^{10}B , ^{11}B) cannot be produced in stars, and their synthesis is believed to be due primarily to CR collisions involving heavier elements (C,N,O). The heavy nucleus that is disrupted may be either the CR projectile itself or the target in the ISM.

Another result of nuclear interactions with CRs is the production of certain radioactive isotopes in the CR population, such as ^{10}Be , ^{26}Al , and ^{36}Cl (with half-lives of hundreds of kyr to a few Myr), which can be observed directly. In addition to providing direct evidence for the synthesis of new nuclei in CR interactions, these nuclei serve as clocks and are useful as probes of CR propagation.

Particles: Collisions of nuclei can produce new particles; in fact many of today's well-known elementary particles were first discovered as products of cosmic ray interactions in the Earth's atmosphere and only later produced in accelerators. We note, however, that most of these particles are unstable, which has obvious implications for CR observables.

One common product of high-energy nuclear collisions is the **pion**, with a mass of $\sim 140 \text{ MeV}/c^2$. Pions come in three types: π^+ , π^0 , and π^- , with the indicated charge; the $\pi^\pm$ are antiparticles of each other, while π^0 is its own antiparticle. The quark content is $\pi^+ = u\bar{d}$, $\pi^- = \bar{u}d$, and $\pi^0 = 2^{-1/2}(u\bar{u} - d\bar{d})$.

The decay mechanisms of the charged and neutral pions are very different. Since pions are the lightest quark-containing particles, they cannot undergo strong decays. The neutral pion can undergo electromagnetic decay:

$$\pi^0 \rightarrow 2\gamma.$$

The photons have energy 70 MeV in the pion rest frame. Since pions are generally not produced at rest, the photons produced may be boosted and form a continuous emission spectrum that peaks at a few hundred MeV. These pion decay photons dominate the luminosity of galaxy in the few hundred MeV to few GeV band, and are principally responsible for the diffuse emission seen by the Fermi satellite.

The charged pion decay chain is:

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e.$$

This produces cosmic ray positrons. These lose energy through the same mechanisms as electrons; eventually they slow down and annihilate against electrons. The muons are short-lived; they can be observed directly only in interactions with the Earth's atmosphere. The neutrinos have also been observed in the context of interactions with Earth's atmosphere; there should also be a diffuse background of neutrinos produced in the ISM (and in the ISMs of other galaxies), but as yet this has not been detected.

Finally, some cosmic ray interactions produce antinucleons, e.g.

$$p + p \rightarrow p + p + p + \bar{p}.$$

This requires a center of mass energy of 2 GeV to produce both the new proton and the antiproton, and hence an incident proton energy of at least 6 GeV. A similar reaction produces antineutrons, which decay to antiprotons:

$$\bar{n} \rightarrow \bar{p} + e^+ + \nu_e.$$

Antiprotons have been observed in the CR population with an antiproton:proton ratio of $\sim 10^{-4}$. Their mode of production requires a very large center of mass velocity and as such imprints a rapid fall-off in the antiproton spectrum at energies of $< \text{few GeV}$; this is in fact observed.

In theory it is possible to produce extremely small quantities of antinuclei (mainly antideuterium, $\bar{D}$), but this has not yet been detected.