Technical Document for Mock LISA Data Challenge 3.5

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We present here our method for finding the stochastic gravitational wave background level and noise levels for Mock LISA Data Challenge 3.5. Our approach is to create a model of the strain spectral densities for the three LISA channels and to fit the data to our model. We calculate the theoretical LISA channel responses to both instrument noise and a stochastic gravitational wave background signal. There are three LISA channels and three unique cross spectra for these channels. Our model consists of these six strain spectral densities.

We fit our predictions for the spectra to our data using the following liklihood function:

$$\mathcal{L} = \frac{1}{(2\pi)^{N/2}|C|} \exp\left(-\frac{1}{2} X_i C_{ij}^{-1} X_j\right) \tag{1}$$

Where C is the noise correlation matrix given by:

$$C = \begin{pmatrix} AA & AE & AT \\ EA & EE & ET \\ TA & TE & TT \end{pmatrix} \tag{2}$$

and $X_i = \{A, E, T\}$ are the MLDC versions of the LISA TDI channels developed in Ref. [1]. We compare the unnormalized posterior density (likelihood × prior density) at one location in parameter space to another using a Hastings Ratio, and use a Parallel Tempered Markhov Chain Monte Carlo search to explore the parameter space and produce a maximum a posteriori (MAP) estimate by finding the highest mode of the posterior distribution.

We derived analytic expressions for the various cross spectra, AA, AE etc for the instrument noise and the stochastic background. Our noise model has a total of 12 parameters - the six position noise levels and the six acceleration noise levels. Our signal model has just one parameter - the overall amplitude $\Omega_{\rm gw}$. We calculate the LISA response to a stochastic background with spectral density

$$S_h(f) = \frac{3H_0^2}{4\pi^2} \frac{\Omega_{gw}(f)}{f^3} \tag{3}$$

However, when we tested our algorithm on the noise-free training data, we found that the injected signal was not actually an f^{-3} spectrum. Therefore, instead of using the analytically calculated transfer functions, we calibrated our transfer functions using the training data.

Using the A,E, and T channels we successfully recover the stochastic gravitational wave background level for the training data. Our recovered values for the blind data are listed in Table 1. We also tested our method using only the X,Y, or Z channels. For an isotropic signal such as a stochastic background, we expect to find symmetry between X,Y, and Z. We find however that the Z channel consistently recovers the background level at a higher value than both X and Y. The average background level from the three channels X,Y, and Z is approximately equal to the recovered level using A,E, and T. We are concerned that there is a systematic error in the data causing this behavior in the Z channel that may skew our results. We found this behavior in the Z channel in both the blind data and training data. However, since the behavior is consistent in the data available to us,and because we calibrated our signal transfer functions from the training data, we believe that we have recovered the proper background level in the blind data despite our concerns over the issue with the Z channel. Figures 1-2 show the recovered background levels for the various

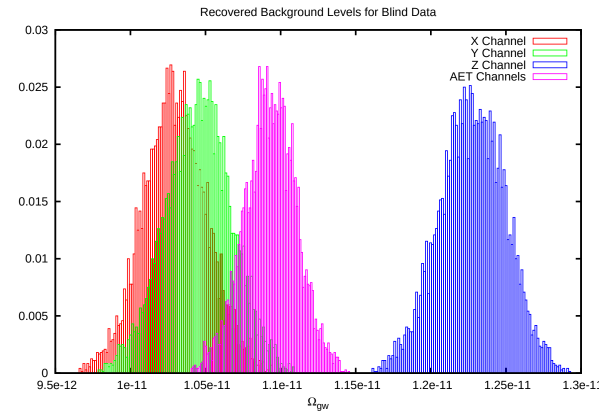

FIG. 1: The recovered background levels for the blind data for challenge 3.5

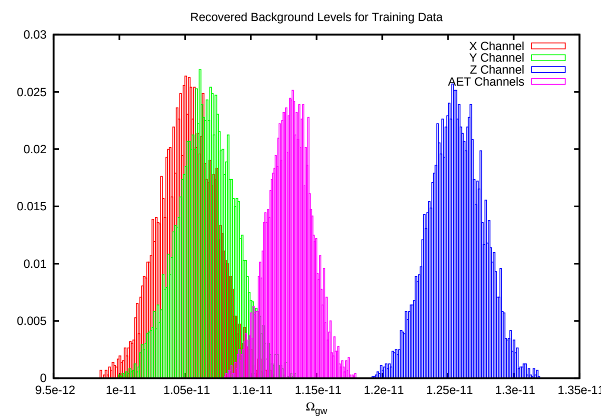

FIG. 2: The recovered background levels for the training data for challenge 3.5

Blind Data Values	
Ω_{gw}	1.088542e-11
pm1	4.545512e-48
pm1s	3.301650e-49
pm2	5.004166e-48
pm2s	1.957746e-48
pm3	2.909435e-48
pm3s	1.907487e-48
pd1	1.200031e-37
pd1s	1.755863e-37
pd2	1.171567e-37
pd2s	2.532888e-37
pd3	1.812443e-37
pd3s	2.633296e-37

LISA channels.

The individual noise parameters are not well constrained. We are only sensitive to the total position or acceleration noise in a particular arm of LISA. The noise combinations that are well constrained are $\{\mathrm{pm1}, \mathrm{pm2s}\}$, $\{\mathrm{pm1s}, \mathrm{pm3}\}$, and $\{\mathrm{pm2}, \mathrm{pm3s}\}$ for acceleration noise and similarly for position noise: $\{\mathrm{pd1}, \mathrm{pd2s}\}$, $\{\mathrm{pd1s}, \mathrm{pd3}\}$, and $\{\mathrm{pd2}, \mathrm{pd3s}\}$. A histograms of the constrained position noise combinations is shown in Figures 3.

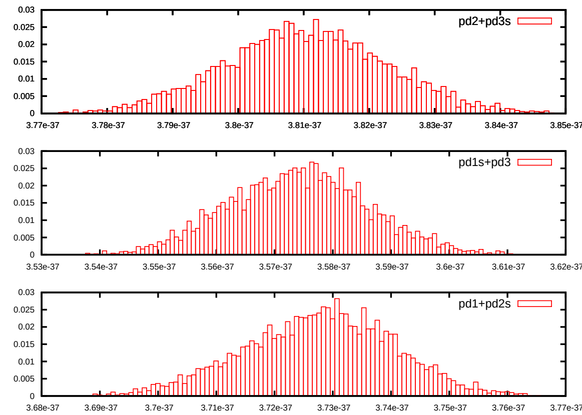

FIG. 3: Constrained Position Noise Combinations

We are currently writing a paper that fully details our method and will soon be submitted.

[1] T.A. Prince, M.Tinto, S.L. Larson J.W. Armstrong, Phys. Rev. D**66**, 122002 (2002).