# Ay 123 Lecture XII Stellar Evolution

## Contraction onto main sequence

- Protostellar clouds start out cool, with low density, and are composed of mainly molecular gas ($H_2$, He, CO, etc).
- Protostar begins to form and contract due to self-gravity
- As it contracts, it heats, beginning to dissociate molecules and ionize atoms:

| | | |
|---|---|---|
| $H_2$ | $E_{Diss} \sim 4.5$ eV | $T \sim 5.2 \times 10^4$ K |
| HI | $E_{ion} \sim 13.6$ eV | $1.6 \times 10^5$ K |
| HeI | 24.6 eV | $2.9 \times 10^5$ K |
| HeII | 54.4 eV | $6.3 \times 10^5$ K |

- When partial ionization begins, $\gamma < 4/3$, protostar is unstable and continues to collapse, heat up
- When gas is ionized, $\gamma \sim 5/3$, contraction slows, protostar shines due to release of gravitational energy

Energy budget:

To dissociate and ionize the gas, the required energy per unit mass is

$$\epsilon = X\left(\frac{E_{ion,H}}{m_p} + \frac{E_{diss,H_2}}{2m_p}\right) + Y\frac{E_{ion,He}}{4m_p}$$

Let $X+Y=1$ (ignore metals), $Y=(1-X)$

$$\epsilon = 1.9 \times 10^{13}\,\frac{\text{erg}}{\text{g}}\,(1-0.2X)$$

Virial therem states

$$\frac{1}{2}\alpha\frac{GM^2}{R} = M\epsilon \qquad (\alpha \sim 6/7 \text{ for } \gamma = 5/3)$$

So, to dissociate/ionize all the gas, the protostar must have

$$\frac{R}{R_\odot} \lessapprox \frac{43.2\,(M/M_\odot)}{(1-0.2X)}$$

Protostars larger are mostly molecular gas, smaller are mostly ionized. At this point, the central temp is

$$\frac{3}{2}\frac{k_B T_c}{\mu m_p} \sim \frac{GM}{R} \sim \epsilon$$

$$\Rightarrow T_c \sim 3\times10^5\,K\ \mu(1-0.2X)$$

So central regions are starting to become ionized, but there is no nuclear burning.

The luminosity of a protostar arises from gravitational energy:

$$L = \frac{dE_{therm}}{dt}$$

$$= \frac{1}{2}\alpha\frac{d}{dt}\left(\frac{GM^2}{R}\right)$$

$$= -\frac{\alpha}{2}\frac{GM^2}{R}\frac{\dot{R}}{R}$$

Protostar collapses on KH timescale:

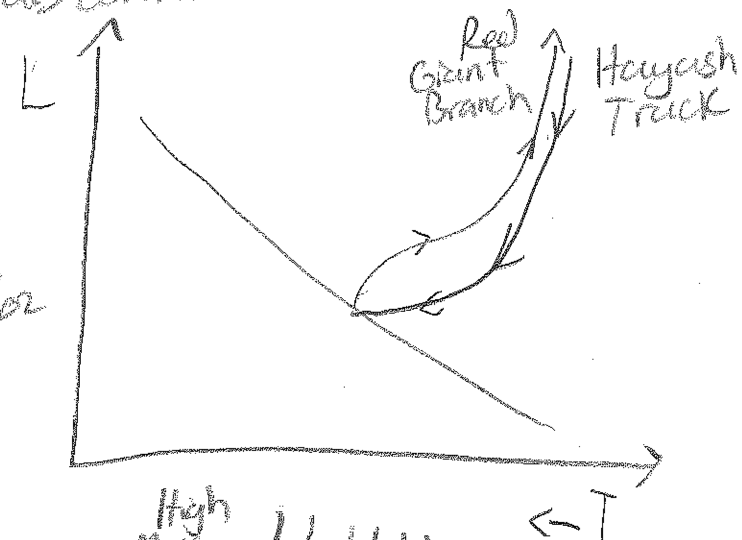
$$t_{col} \sim \frac{GM^2}{RL}$$

$$\sim 1.6\times10^7\,yr\left(\frac{M}{M_\odot}\right)^2\left(\frac{R}{R_\odot}\right)^{-1}\left(\frac{L}{L_\odot}\right)^{-1}$$

Protostars are almost totally convective due to partial ionization.

⇒ Nearly totally convective ideal gas, $\gamma \simeq 5/3$
⇒ $H^-$ opacity at surface
⇒ Very similar surface properties to red giants
⇒ Red giants expand, protostars contract

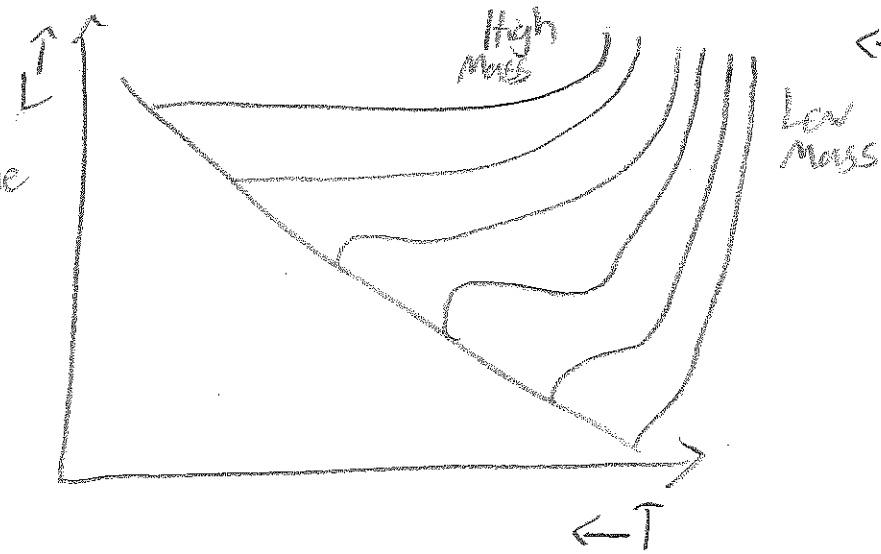
- Similar envelope structures, very different core structures




Hayashi track

$$T_{ef} \simeq 4000\,K\,\mu^{13/51}\left(\frac{M}{M_\odot}\right)^{7/51}\left(\frac{L}{L_\odot}\right)^{1/102}$$

- Pre-MS evolution like Post-MS evolution in reverse
- Occur on different time scales ($\tau_{therm}$ for Pre-MS, $\tau_{nuc}$ for Post-MS)




## Main Sequence Evolution

- Zero age main sequence (ZAMS) occurs when H-burning begins, $T_c \gtrsim 10^7\,K$

Low-mass stars ($M \lesssim 1.3\,M_\odot$)
- PP chain, radiative core, convective envelope

High-mass stars
- CNO burning, convective core, radiative envelope

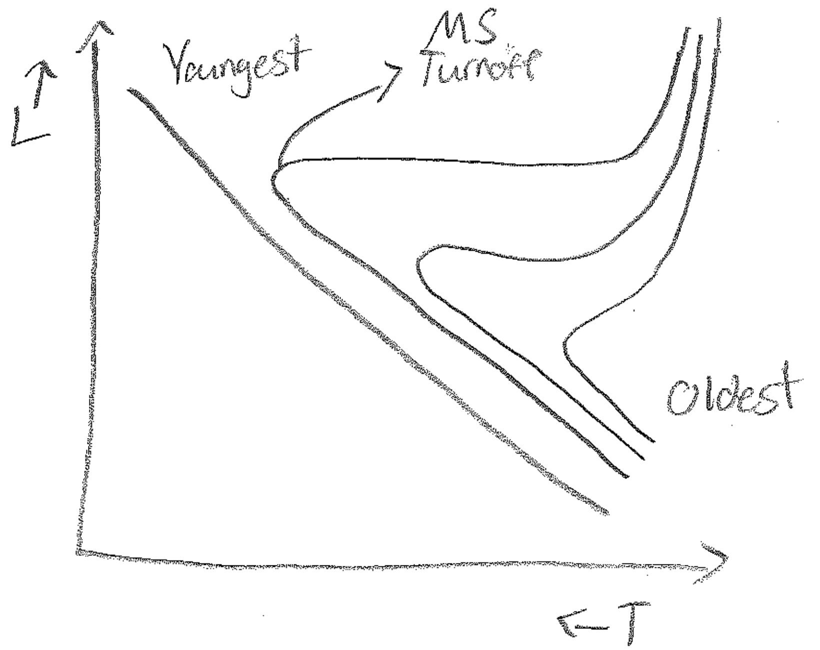
- MS evolution slow

$$\tau_{nuc} \sim \frac{\Delta M c^2}{L} \gg \tau_{therm}$$

- Structure hardly changes
- determining stellar ages is difficult!
- $\epsilon \propto T^{4-17}$, so H is depleted first at center of star
- H then burned in shell around He core
- Because $\tau_{MS} \propto \frac{M_{core}}{L} \propto M^{-3}$, clusters of different age look very different

☆ - Isochrones can be fit to cluster HR diagram to determine age.

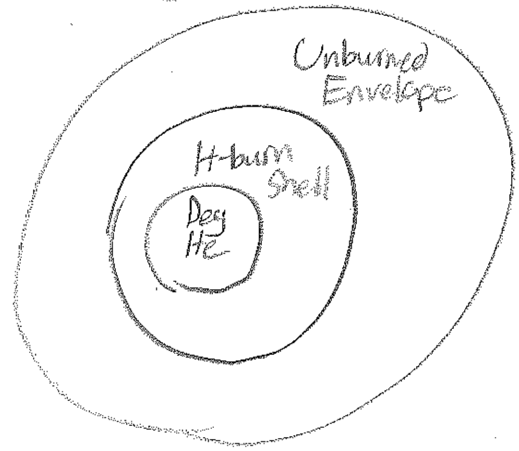
- Stars at MS turnoff have $\tau_{MS} = \tau_{cluster}$




## Post-MS evolution

- When H exhausted in core, thermal pressure support lost
- Core contracts, releases gravitational energy on thermal time
- Star crosses HR gap on thermal time $\sim 10^{4-8}$ yr

- core contraction halted by onset of degeneracy pressure
- Outside of core, H burns in shell, providing thermal support for rest of star




Position of H-burning shell:

- He core is essentially a He white dwarf
- H-burning shell sits at surface of He WD
- $R_{WD} \propto M_{WD}^{-1/3}$

$$\Rightarrow R_{shell} \sim 0.013 R_\odot \left(\frac{M_{core}}{M_\odot}\right)^{-1/3}$$

For low-mass stars, $M_{core} \lesssim 0.4 M_\odot$, $R_{shell} \sim 0.02 R_\odot$

- Core is nearly isothermal (can't cool below $T_{shell} \gtrsim 2\times10^7 K$)
  - $\hookrightarrow$ $e^-$ degeneracy $\Rightarrow$ long $e^-$ mean free path, $e^-$ become good heat conductors, WD cores usually iso-thermal

- As core grows, its radius slighty shrinks, and its pressure grows

$$P_{core} \sim \frac{G M_{core}^2}{R_{core}^4} \propto M_{core}^{10/3}$$

- At burning shell, H is ideal gas,

$\Rightarrow T_{shell}$, $P_{shell}$ increase with $M_{core}$

$\Rightarrow L_{shell}$ increase sharply with $L_{core}$

$\Rightarrow R_{surf}$ increase (because surface temp determined by $H^-$ opacity, $T_{eff} \sim 4000 K$)

- Red giant expands and brightens as core grows
- Less time spent high on RGB

$$\tau \propto L^{-1}$$

# Helium Burning

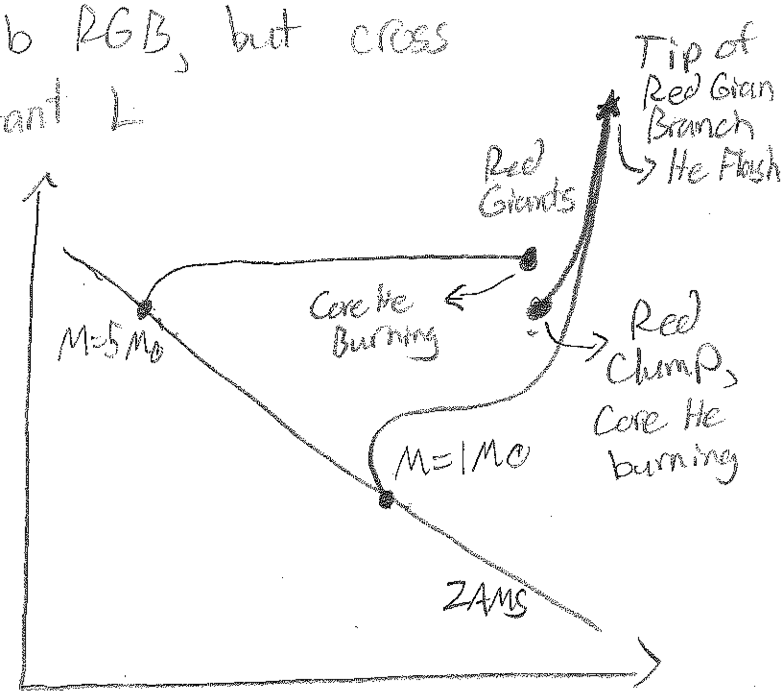
- He burning via triple-$\alpha$ begins when core reaches ~$10^8$ K
- For low-mass stars ($M \lesssim 2.5 M_\odot$), He core not massive/hot enough to ignite after MS
  - Hydrogen shell-burning builds He core to ~$0.5 M_\odot$
  - He burning triggered in degenerate He core

He Flash
  - For degenerate core, P independent of T
  - Nuclear burning increases T, which increases burning rate
  - Produces runaway, which slows once T becomes large enough to lift degeneracy, expand/cool core

  - Star settles into equilibrium with core He burning, shell H-burning

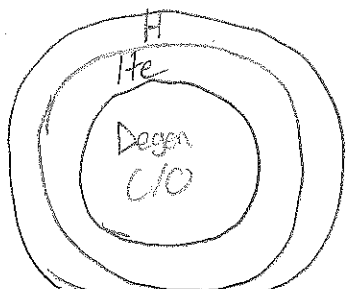
- For high-mass stars ($M \gtrsim 2.5 M_\odot$), He core ignites upon contraction after MS
  - He flash avoided
  - These stars do not climb RGB, but cross HR gap at nearly constant L

- Very low-mass stars ($M \lesssim 0.6 M_\odot$) cannot trigger He burning, evolve into He White Dwarfs
  ↳ Takes longer than age of universe

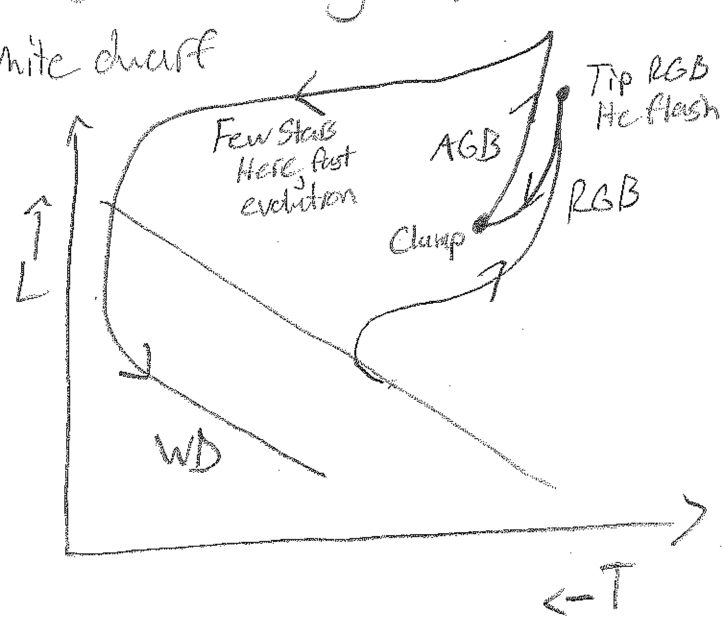


# Carbon burning and beyond

- Stars with $M \lesssim 6\,M_\odot$ do not build C/O cores massive enough to ignite C-burning at $T \simeq 6 \times 10^8\,K$.
  - C/O core contracts to become degenerate
  - He shell burning, H shell burning
  - Star ascends asymptotic giant branch (AGB)
  - Shells burn outward, mass is lost from surface, until shells nearly reach surface and burning stops
  - Star then become C/O white dwarf




$M_{WD} \sim 0.6\,M_\odot$




- Stars with $6\,M_\odot \lesssim M \lesssim 8\,M_\odot$ ignite C-burning, but not Ne burning
  - Produce O/Ne white dwarfs of $\sim 1.2\,M_\odot$

- Stars with $M \gtrsim 9\,M_\odot$ ignite Ne burning, O burning, Si burning
  - Produce core-collapse SN, neutron star
- Stars with $M \sim 8.5\,M_\odot$ do not ignite Ne burning, O/Ne core becomes dense enough to begin neutronization, which remove $e^-$ degeneracy pressure, causes core-collapse
  - Electron capture SNe

# Mass - Loss

- Negligible for low-mass stars ($\dot{M} \sim 10^{-14} M_\odot/yr$ for Sun)
- Becomes very important for luminous stars
- Mass driven out by radiation pressure
  - Opacity due to absorption lines in hot stars ($T \gtrsim 10^4 K$)
  - Opacity due to molecules/dust in cool stars ($T \lesssim 4000 K$)
- Hard to predict, calibrated mass loss rates used

$$\dot{M} \propto L^\alpha T^\beta$$

- Important at tip AGB, where hydrogen envelope lost in low-mass stars
- Important on MS for very massive ($M \gtrsim 30 M_\odot$) stars which near Eddington limit

## Eddington limit

- Occurs when gravity balanced by radiation pressure

Hydrostatic equilibi

$$\frac{\partial P}{\partial r} = -\rho g$$

For radiation pressure, $P = \frac{1}{3} a T^4 \Rightarrow \frac{\partial P}{\partial r} = \frac{4}{3} a T^3 \frac{\partial T}{\partial r}$

$$= \frac{-K\rho}{c} F_{rad}$$

$$\Rightarrow \frac{K F_{rad}}{c} = g$$

$$\Rightarrow L_{ed} = 4\pi r^2 F_{rad} = \frac{4\pi r^2 g c}{K}$$

$$\Rightarrow \boxed{L_{ed} = \frac{4\pi G M c}{K}}$$

$$L_{ed} \simeq 3 \times 10^4 L_\odot \left(\frac{M}{M_\odot}\right)$$

for Thomson scattering opacity

- Eddington limit is maximum radiative luminosity for spherically symmetric star in hydrostatic equilib
- Can be exceeded if:
  - Convection carries luminosity
  - Aspherical emission
  - Star is not hydrostatic
  
  $L > L_{Ed}$ typically leads to stellar outflows

- Stars with $M \gtrsim 30\,M_\odot$ approach Eddington limit, lose most of hydrogen envelope via winds
  - Evolve into hydrogen-free He burning Wolf-Rayet stars