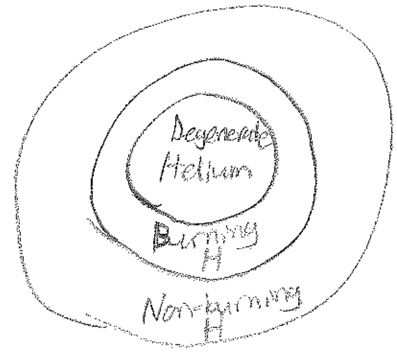
# Ay 123 Lecture XI Supernovae and Neutron Stars

## End of hydrostatic nuclear burning:

After a phase of nuclear burning, stars often develop degenerate cores with no nuclear burning, surrounded by a burning shell.




If mass of degenerate core increases enough, its pressure/temperature become high enough to ignite next burning phase. Otherwise, a degenerate star (white dwarf) is formed.

Stars with $M \gtrsim 10\,M_\odot$ continue this cycle of core burning/contraction until they have built a core of $^{56}Fe$.

In a $20\,M_\odot$ star,

$H \to He \quad \sim 10^7\,yr$

$He \to C \quad \sim 10^6\,yr$

$C \to O + Ne \quad \sim 10^3\,yr$

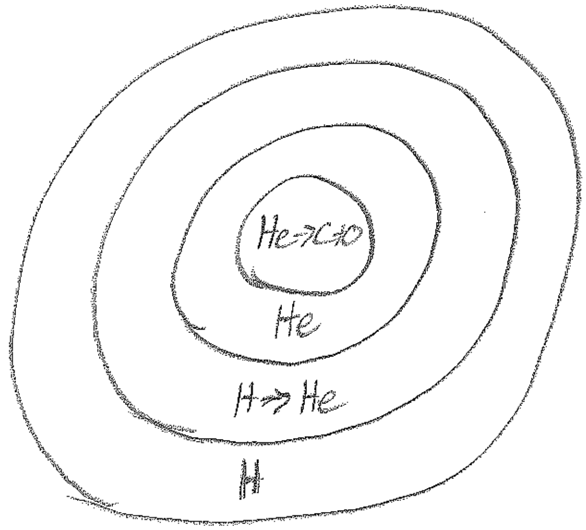
$Ne \to O + Mg + Si \quad \sim 10\,yr$

$O \to Si + S + \text{Other} \quad \sim 1\,yr$

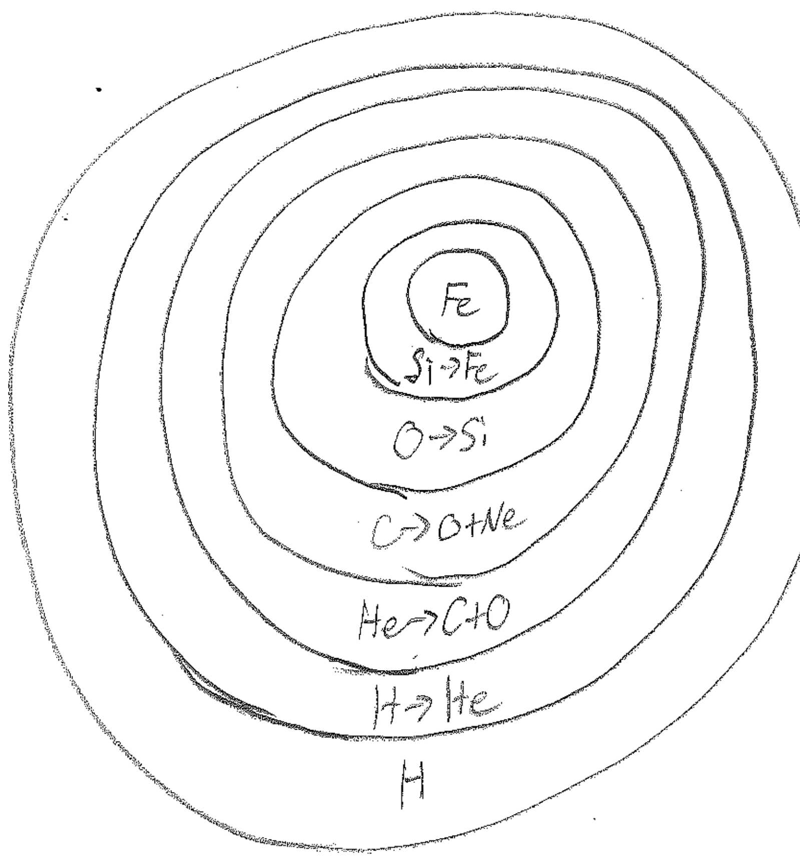
$Si \to Fe + \text{Other} \quad \sim 1\,\text{week}$

Reaction rates increase (and burning times decrease) because later reactions release less energy per mass, and because neutrinos carry away an increasing fraction of nuclear fusion energy.

Many phases of burning can proceed simultaneously.
In low-mass red clump star:




In pre-collapse 20 M☉ star:



# Core-collapse

When iron core reaches $M_{Fe} \gtrsim M_{ch} \approx 1.4\,M_\odot$, it begins to collapse. As it collapses, it heats up to $T \sim 10^{10}$ K.
At these temperatures, photons begin to photodisintegrate iron:

$$^{56}Fe \rightarrow 13\,^4He + 4n - 124\text{ MeV}$$

Loss of photon pressure causes core-collapse to accelerate! Iron core collapses into neutron star! Collapse occurs on dynamical timescale of iron core:

$$\tau_{col} \sim \sqrt{\frac{R_{core}^3}{GM_{ch}}} \sim 10^{-1}\text{ s}$$

Binding energy of collapsing core is $\sim 10^{52}$ erg. Some of this is used to convert $^{56}Fe \rightarrow {}^4He$.
Photons of 124 MeV entails temperatures $T \sim 10^{12}$ K.
However, such large $T$ not actually required because some high $E$ photons exist in tail of energy distribution. Boltzmann distribution can be used to derive nuclear "Saha" equation:

$$\frac{n_\alpha^{13} n_n^4}{n_{Fe}} \sim \frac{g_\alpha^{13} g_n^4}{g_{Fe}} \left(\frac{C_\alpha^{13} C_n^4}{C_{Fe}}\right) e^{-124\text{ MeV}/k_BT}$$

where $$C_x = \left(\frac{2\pi m_x k_B T}{h^2}\right)^{3/2}$$

and $g_\alpha = g_{Fe} = 1$, $g_n = 2$.
At $T \sim 10^{10}$ K, $\rho \sim 10^9$ K, 75% of $^{56}Fe$ disintegrated.
This requires $\sim 3 \times 10^{51}$ erg per $M_\odot$.

# Neutronization

Neutrons more massive than protons:

$$(m_N - m_p)c^2 = 1.3 \text{ MeV}$$

Free neutrons decay:

$$n \rightarrow p + e^- + \bar{\nu}_e$$

But neutrons produced by $e^-$ capture:

$$p + e^- \rightarrow n + \nu_e$$

In dense core where $e^-$ are degenerate, neutron decay is prevented because $e^-$ Fermi states are filled ($e^-$ has large chemical potential). As density increases, $e^-$ capture rate increases. So $p + e^-$ are converted to $n$.

Neutronization begins when Fermi energy is sufficient to create neutron rest mass:

$$E_e = \left(p_F^2 c^2 - m_e^2 c^4\right)^{1/2}$$

$$\Rightarrow p_F^2 c^2 = \underbrace{E^2}_{1.3 \text{ MeV}} - (m_e c^2)^2$$

$$\Rightarrow p_F \sim 2.2\, m_e c$$

Using $n_e = \frac{8\pi}{3h^3} p_F^3$, $\rho = \mu_e m_p n_e$

$$\Rightarrow \rho \gtrsim 2.4 \times 10^7 \text{ g/cm}^3$$ Neutronization begins

In realistic situations, neutronization begins through neutronization of atomic nuclei. Neutron-rich elements are synthesized in dense environments.

At high enough densities, atomic nuclei begin to dissolve into neutrons, and small fractions of $p + e^-$. This is called neutron drip, and occurs when $\rho \gtrsim 4 \times 10^{11}\, g/cm^3$. See KWW § 16.5

To convert $^{56}Fe$ to pure neutrons requires a lot of energy!

$$E = \Delta m\, c^2$$

$$= 8 \times 10^{-4}\ erg/\text{Fe Nucleus}$$

$$= 2 \times 10^{52}\ erg/M_\odot$$

Binding energy of iron core is not sufficient to supply this energy. However, binding energy of neutron star is

$$E_{NS} \sim \frac{G M_{NS}^2}{R_{NS}} \sim 5 \times 10^{53}\ erg$$

Hence, some of gravitational binding energy released by collapse to NS is used to convert $^{56}Fe / ^4He$ into neutrons.

Neutrino production

During neutronization, neutrinos (and anti-neutrinos) produced by reactions

$$\left.\begin{aligned} p + e^- &\rightarrow n + \nu_e \\ n &\rightarrow p + e^- + \bar{\nu}_e \end{aligned}\right\}\ \text{Urca process, neutrino cooling}$$

The number of neutrinos produced is at least

$$N_{\nu_e} \sim N_N \sim \frac{M_{NS}}{m_N} \sim 10^{57}$$

This is a very big number even in astrophysics. Typical neutrino energies are ~10 MeV. Energy radiated in neutrinos is at least

$$E_{\nu,tot} \sim N_{\nu_e} E_\nu$$
$$\sim 10^{58}\ \mathrm{MeV} \sim 10^{53}\ \mathrm{erg}$$

In reality, nearly all of the neutron star binding energy is radiated in neutrinos! The proto-NS radiates $\sim 5\times10^{53}$ erg of $\nu_e$ over the first few seconds of its life. The typical supernova explosion contains only $\sim 10^{51}$ erg of energy, less than 1% of the neutrino energy radiated.

## SN 1987A

- Nearest supernova since 1604
- In Large Magellanic Cloud, $D \sim 50$ kpc
- 25 neutrinos detected on Earth
- Detected ~3 hours before photons from SN

## Supernova Explosion mechanism (core-collapse)

- Massive star ($M \gtrsim 10 M_\odot$) evolves until Fe core exceeds $M_{ch}$.
- Photodisintegration/neutronization expediate collapse
- When core reaches nuclear densities ($\sim 10^{14}$ g/cm$^3$), EOS stiffens due to strong force repulsion, degeneracy pressure of neutrons

- Core compresses to super-nuclear densities, "bounces" back and launches shock wave into infalling material

- Shock wave energy used to dissociate $^{56}Fe \rightarrow {}^{4}He$, and ram pressure of infalling material causes shock to stall

- In order for successful explosion, shock must be "revived" by some mechanism

- Neutrino mechanism is most popular explanation
- Material below shock absorbs some of radiated $\nu$ energy, rejuvinating it, and driving SN explosion of the rest of the star
- Explosion occurs on dynamical time scale of stellar core ($\sim 1s$) before the rest of the star has had time to collapse ($\tau_{dyn,surf} \gtrsim 1 hr$).

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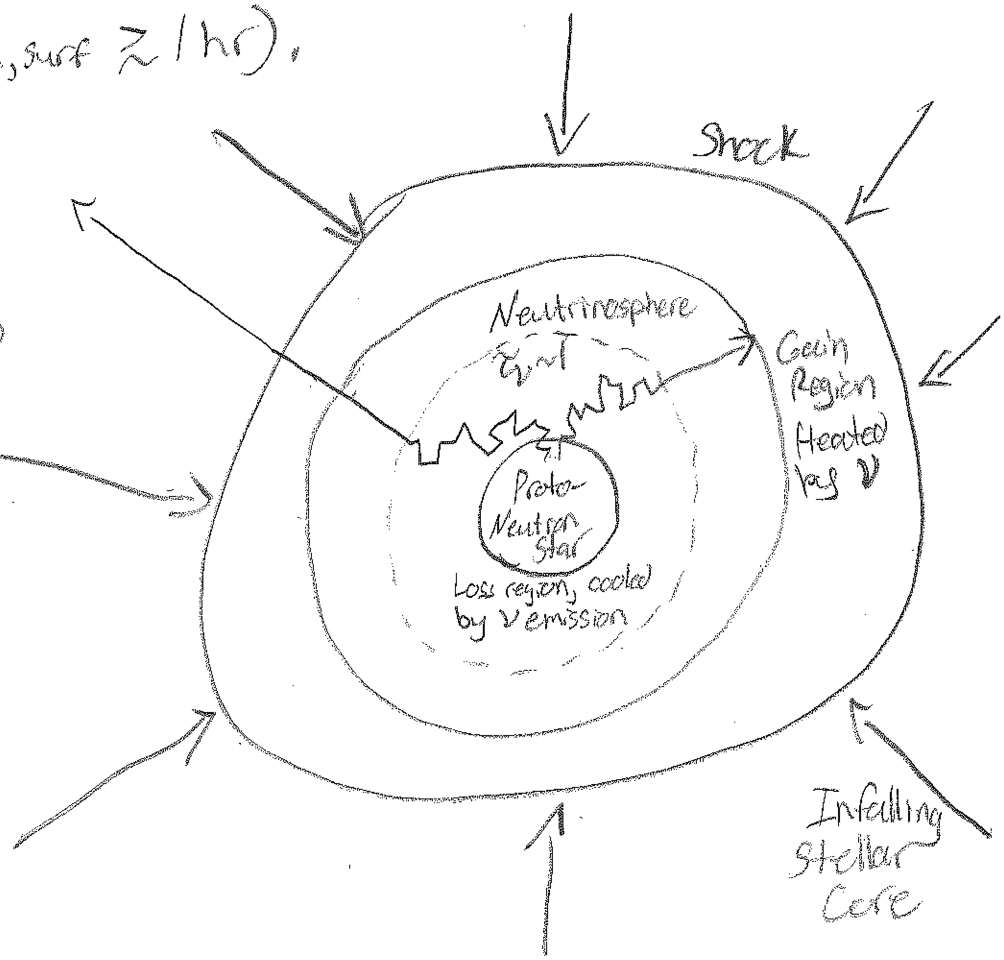
Neutrino mechanism

- Shock revived through absorption of neutrino emission in "gain" region below shock



# Neutrino Diffusion

At high densities, neutrinos will scatter off nucleons many times before escaping the proto-NS! Neutrinos diffuse out over a thermal time (~few seconds). The neutrino cross section is

$$\sigma_\nu \sim \left(\frac{E_\nu}{m_e c^2}\right)^2 1.7\times10^{-44}\ cm^2$$

Neutrinos typically have energies

$$E_\nu \sim E_F \sim p_F c$$

$$\Rightarrow \sigma_\nu \sim \left(\frac{\rho}{\mu_e}\right)^{2/3} 10^{-49}\ cm^2$$

So the neutrino mean free path is

$$l \sim \frac{1}{n_N \sigma_\nu} = \frac{m_N}{\rho \sigma_\nu}$$

$$\Rightarrow l \sim \mu_e^{2/3} \rho^{-5/3}\ 10^{25}\ cm$$

$$\Rightarrow l \sim 2\ Km \left(\frac{\rho}{10^{12}\ g/cm^3}\right)^{-5/3}$$

At center of neutron star, $\rho \sim 3\times10^{14}\ g/cm^3$, and mean-free path is much smaller than NS radius! Density falls in overlying star, and the point where the overlying optical depth $\tau_\nu = \int \frac{dr}{l} = 1$ is the "neutrino sphere", typically at $\rho \sim 3\times10^{11}$ in a core-collapse SN.

Above neutrinosphere, only small fraction of $\nu$ interact. Simulations show that ~10% of neutrino energy can be absorbed in gain layer. This layer is cool and low density such that it does not generate its own neutrinos, so the absorbed energy can be used to heat up the gas below the shock. The hot gas expands, pushing the shock outward. If enough energy is deposited, shock can propagate into low-density material, and accelerate out, depositing $\sim 10^{51}$ erg of thermal energy, blowing up overlying star.

Material behind shock is very hot, and many nuclear reactions occur. Core-collapse SN generally produce large amounts of $\alpha$ elements (C, O, Ne, Ar, Ca, Mg, etc) during explosion.

<u>Relativistic Effects</u>

From Einstein's equivalence principle, both rest-mass and energy gravitate. The density of gravitational mass is

$$\rho = \rho_0 + \frac{U}{c^2}$$

$\frac{U}{c^2}$ ~> Kinetic Energy Density

$\rho_0$ ↳ rest mass

For neutron gas,

$$\rho = N_N m_N + \frac{U}{c^2}$$

For relativistic $e^-$ gas, Kinetic energy density of $e^-$ exceeds the $e^-$ rest mass density, but is usually much smaller than ion rest mass density.

For relativistic neutron gas,

$$\rho_0 \lessapprox \frac{U}{c^2} \Rightarrow \rho \simeq \frac{U}{c^2}$$

$$\Rightarrow P = \frac{U}{3} \simeq \frac{\rho c^2}{3} \Rightarrow P \propto \rho$$

$$\Rightarrow \gamma = 1 \text{ polytrope, unstable!}$$

This reduces maximum possible NS mass.
For NS, the baryonic and gravitational masses are <u>different</u>.

$$M_{bary} = N_N m_N$$

$$M_{grav} = \frac{E_{tot}}{c^2} = \frac{E_{rest}}{c^2} + \frac{E_{kin}}{c^2} + \frac{E_{bind}}{c^2}$$

From virial theorem,

$$E_{kin} = -\frac{1}{2} E_{bind}$$

$$E_{bind} \sim \frac{-GM_{bary}^2}{R}$$

So

$$M_{grav} \simeq \frac{E_{rest}}{c^2} - \frac{1}{2}\frac{GM_{bary}^2}{Rc^2}$$

$$\simeq M_{bary}\left(1 - \frac{GM_{bary}}{2Rc^2}\right)$$

$$\simeq M_{bary}\left(1 - \frac{R_S}{4R_{NS}}\right)$$

Swarzschild Radius $R_S = \frac{2GM}{c^2}$

Gravitational mass of NS is <u>smaller</u> than its baryonic mass by $\sim 10\%$.

## Gravitational Redshift

Photons escaping from surface of NS must escape deep gravitational potential.

Photon energy at NS surface

$$E_{NS} \simeq h\nu_{NS}\left(1 - \frac{GM_{NS}}{Rc^2}\right)$$

→ Gravitational Energy

At infinity

$$E_\infty = h\nu_\infty$$

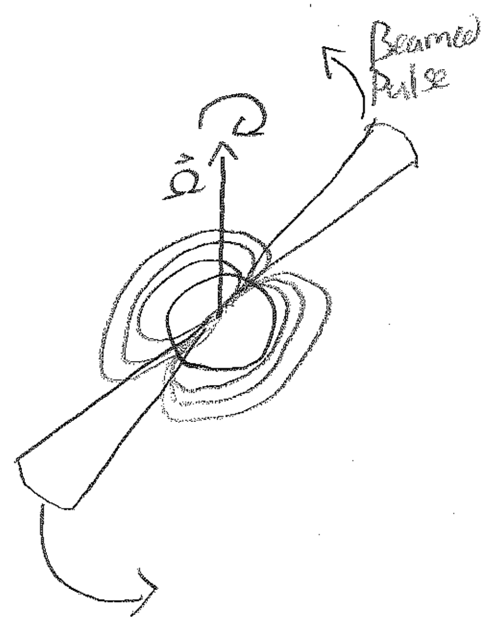
In full GR,

$$\nu_\infty = \nu_{NS}\left(1 - \frac{R_S}{R_{NS}}\right)^{1/2}$$

Escaping photons have lower frequency/energy because they have to do work to escape gravitational field.

## Pulsars

- Rotating NS emit radio pulse in beams along magnetic axis
- We see a pulse of radio every spin period
- Millisecond pulsars have $P_{spin} \sim$ few ms
- Timing of pulses in binary system can be used to measure mass of pulsars
- Pulse timing can be used to detect GW
- Pulsars spin too fast to be white Dwarfs




☆ $$\Omega_{max} \simeq \left(\frac{GM}{R^3}\right)^{1/2}$$

$$\Rightarrow P_{min} \simeq t_{dyn} \simeq 1\,s \text{ for WD}$$

$$\sim 1\,ms \text{ for NS}$$