Ay 123 Lecture IV. Energy generation and opacity

Energy generation $\epsilon(\rho, T)$ from nuclear physics

Opacity $\kappa(\rho, T)$ from atomic physics

4.1 Nuclear Physics

Relatively "new" physics, more recent than relativity!

1932: Neutrons discovered by Chadwick

Fission produced by Rutherford, $^7Li + p \rightarrow 2\,^4He$

1938: Bethe works out PP-I, PP-II burning chains

Exothermic nuclear reactions convert mass to energy

$$E_{Nuc} = \Delta m c^2$$

$$= (\text{Mass of reactants} - \text{Mass of products})\, c^2$$

N = neutron #

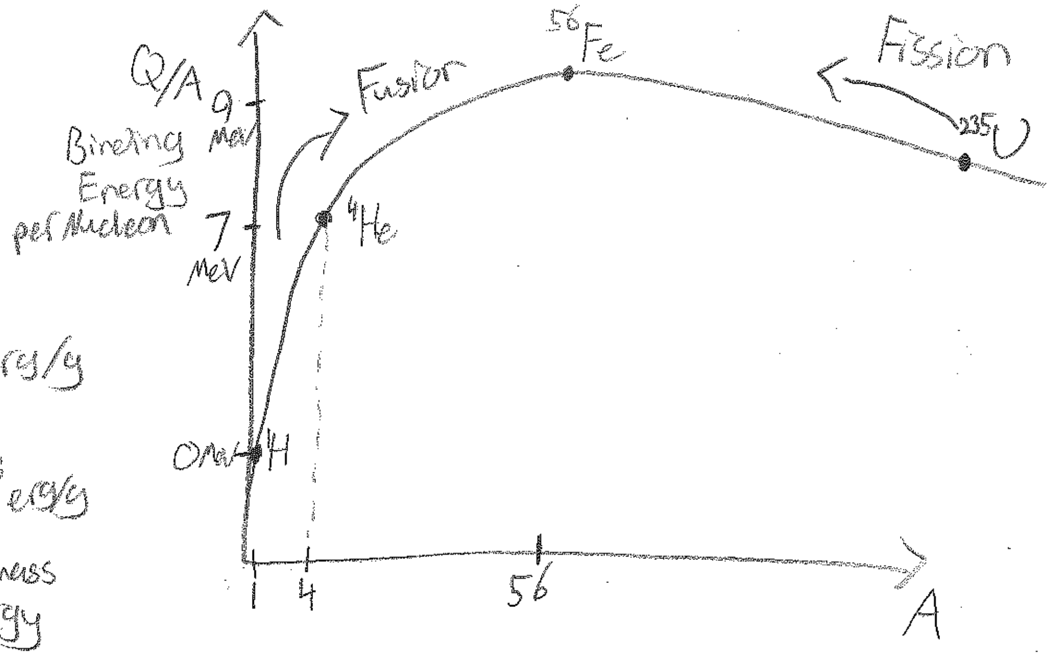
Z = proton #

A = atomic # $= N + Z$

Q = binding energy of nucleus

$$= \left(Z m_p + N m_N - m_{Nuc}(Z, N) \right) c^2$$

Energy released:



$^{1}H \to {}^{4}He$ $\quad 6.3 \times 10^{18}$ erg/g

$^{1}H \to {}^{56}Fe$ $\quad 7.6 \times 10^{18}$ erg/g

~1% rest mass energy

Weak interactions often important, involve fermions

$$\left.\begin{array}{l} n \to p + e^- + \bar{\nu}_e \rightsquigarrow \text{anti-neutrino} \\ p \to n + e^+ + \nu_e \rightsquigarrow \text{neutrino} \end{array}\right\} \beta\text{-decay}$$

<u>PP-I Chain</u> occurs at $\sim 10-14 \times 10^6$ K

Most important nuclear reaction chain in Sun

1. $p + p \to \overset{\text{di-proton}}{{}^{2}He} + \gamma$
   $\to \overset{\text{deuteron}}{{}^{2}H} + e^+ + \nu_e$ $\quad$ } $+0.42$ MeV
   $\quad \hookrightarrow e^+ + e^- \to 2\gamma$ $\quad$ 1.02 MeV
2. $^{2}H + p \to {}^{3}He + \gamma$ $\quad$ 5.49 MeV
3. $^{3}He + {}^{3}He \to {}^{4}He + 2p$ $\quad$ 12.86 MeV

$$\text{Total} = 2[0.42 + 1.02 + 5.49] + 12.86 = 26.73 \text{ MeV}$$

2% of energy goes into neutrinos

PP—II Chain Occurs at $1.4-2.3 \times 10^7$ K

1. Same first two steps as PP-I
2. $^3He + ^4He \rightarrow ^7Be + \gamma$
3. $^7Be + e^- \rightarrow ^7Li + \nu_e + 0.861$ MeV
4. $^7Li + p \rightarrow 2\,^4He$

PP—III Chain Occurs at $T > 2.3 \times 10^7$ K

1. Same first 2 steps as PP-II
2. $^7Be + p \rightarrow ^8B + \gamma$
3. $^8B \rightarrow ^8Be + e^+ + \nu_e$ ~> -High energy neutrino can be detected
   -Revealed solar neutrino problem
4. $^8Be \rightarrow 2\,^4He$

PP Chains important on lower main sequence, release

$$Q \sim 26.2 \text{ MeV per } ^4He$$

First step of PP-I chain is extremely rare! It essentially requires one of the protons to β-decay (weak, slow reaction) at same moment it collides with other proton. The cross section is thus small, and the reaction rate is very slow, of order 10^{10} years at center of Sun!

CNO Cycle

Occurs at $T \gtrsim 1.5 \times 10^7$ K

- Important for stars with $M \gtrsim 1.2\ M_\odot$

1. $^{12}C + p \rightarrow {}^{13}N + \gamma$ 1.95 MeV
2. $^{13}N \rightarrow {}^{13}C + e^+ + \nu_e$ 1.2 MeV
3. $^{13}C + p \rightarrow {}^{14}N + \gamma$ 7.54 MeV
4. $^{14}N + p \rightarrow {}^{15}O + \gamma$ 7.35 MeV
5. $^{15}O \rightarrow {}^{15}N + e^+ + \nu_e$ 1.73 MeV
6. $^{15}N + p \rightarrow {}^{12}C + {}^4He$ 4.96 MeV

$$Q = 25.0 \text{ MeV per } {}^4He$$

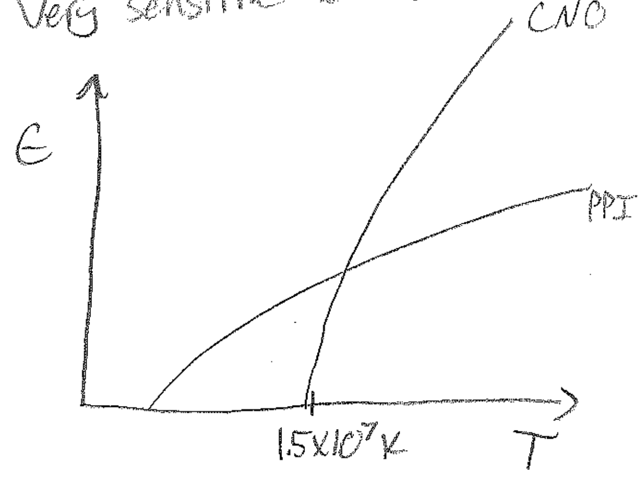
CNO are all catalysts, net reaction is $4p \rightarrow {}^4He + 2e^+ + \nu_e + \gamma$

Reaction rates are complicated!
For our purposes, scaling is most important.

$$\epsilon_{PP\text{-}I} \propto X_H^2 \rho T^4$$

$$\epsilon_{CNO} \propto X_H X_C \rho T^{17}$$

Very sensitive to T!



Helium burning

Next stable nucleus with larger binding energy after ^4He is ^{12}C. But it takes 3 ^4He to make ^{12}C. How can this occur? Not solved until 1950s.

Triple-α process   Occurs at $T \approx 10^8 K$

$^4He + ^4He \rightarrow ^8Be$ -91.8 keV Endothermic!

$^8Be + ^4He \rightarrow ^{12}C + \gamma$ 7.37 MeV

Problem: ^8Be decays with half-life of 7×10^{-17} s. So ^8Be is almost always destroyed before it can fuse with ^4He to make ^{12}C. So, <u>triple</u>-alpha process requires essentially simultaneous collision of 3 ^4He nuclei.

Triple-α process enabled by resonance: ^{12}C has an excited nuclear state very near the rest mass of $^4He + ^8Be$. This means $^8Be + ^4He$ reaction rate is greatly enhanced. Otherwise, helium fusion would not occur and carbon-based life would not exist! Hoyle predicted ^{12}C resonance, Fowler found it experimentally and got Nobel prize.

Rate

$$\epsilon_{He} \propto X_{He}^3 \rho^2 T^{40} \rightarrow \text{Very temperature sensitive}$$

Result of requirement of 3-body collision

Elements made by adding ^4He to ^{12}C are alpha-process elements. They are common because these elements are usually stable and easy to form via nucleosynthesis

$$^{12}C,\ ^{16}O,\ ^{20}Ne,\ ^{24}Mg,\ ^{28}Si,\ ^{32}S,\ ^{36}Ar,\ ^{40}Ca,\ ^{44}Ti,\ ^{48}Cr,\ ^{52}Fe$$

and lastly, ^{56}Ni

Heavier elements have lower binding energies, rarely produced. Note, ^{56}Ni decays to ^{56}Fe with half life of 6 days. This decay powers optical emission of many supernovae.

4.2 Opacity

Recall

$$\frac{dT}{dr} = -\frac{3 K L_{rad} \rho}{16\pi a c r^2 T^3}$$

where K is mean opacity, integrated over all frequencies. In reality, $K = K_\nu$ is function of photon frequeny ν.

Sources of opacity: photon interacting with electron

Bound-bound: Exciting electron from one enery level to another. Occurs at set of discrete frequencies. Low T

Bound-free: Ionization of electron from atom. Occurs at continuous frequencies above ionization threshold $\nu = E_{ion}/h$ Med T

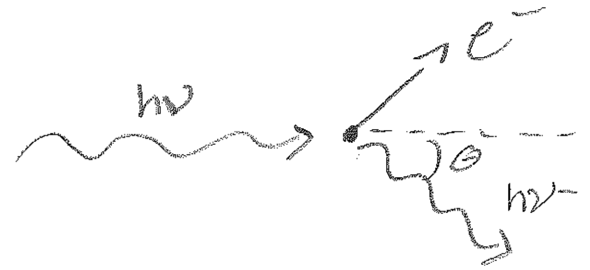
Free-Free : Interaction between ion, free e^-, and photon Med T

e^- scattering: Thomson scattering / Compton scattering
Interaction with free electron
Nearly independent of photon frequency, density, and T

High T

Thomson scattering

Change in photon wavelength is



$$\Delta\lambda = 2\lambda_c \sin^2(\theta/2)$$

where

$$\lambda_c = \frac{h}{m_e c} = \text{Compton wavelength} = 2.4\times10^{-10}\ \text{cm}$$ Short wavelength

Note the change in photon energy is

$$\Delta E = \Delta\left(\frac{hc}{\lambda}\right) = -E\frac{\Delta\lambda}{\lambda} \sim E\frac{\lambda_c}{\lambda}$$

In stellar interiors, $\lambda \sim \frac{hc}{E} \sim \frac{hc}{k_B T} \sim 10^{-7}\ \text{cm} \gg \lambda_c$

So scattering is essentially elastic, photon energy hardly changed.
At higher energies, scattering is inelastic ⇒ Compton scattering.

The cross section, i.e, effective target size of an electron producing Thomson scattering is

Fine structure constant

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 = \frac{8\pi}{3}\left(\frac{\alpha\hbar}{m_e c}\right)^2 = 6.7\times10^{-25}\ \text{cm}^2$$

The corresponding opacity is

$$K_T = \frac{\sigma_T n_e}{\rho}$$

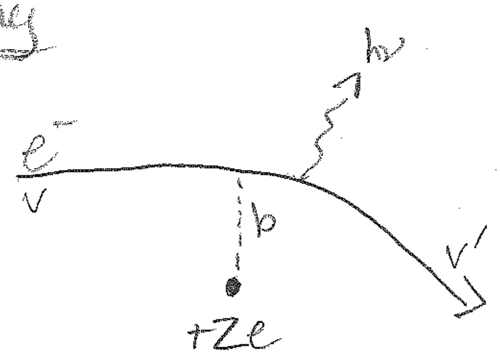
For completely ionized material, this evaluates to

$$\boxed{K_T \approx 0.2\ (1+X)}\ \frac{cm^2}{g}$$

X ↳ hydrogen mass fraction

Note $\sigma_T \propto m_e^{-2}$. Scattering off ions occurs, but is smaller by factor $(m_p/m_e)^2 \sim 10^6$, so is negligible.

Free-free Absorption / Bremsstrahlung



An electron accelerating in electric field of an ion must absorb/emit a photon

Conserving energy,

$$\frac{1}{2} m_e v^2 + h\nu = \frac{1}{2} m_e v'^2$$

The absorbed/emitted energy is

$$E_{abs} = \int_{-\infty}^{\infty} \frac{dE}{dt}\, dt$$

↳ radiation due to accelerating charge

$$= \frac{2e^2}{3c^3} \int_{-\infty}^{\infty} a^2\, dt$$

The acceleration occurs over time of closest approach,

$$t \sim b/v'$$

And the acceleration from Coulomb force is

$$a \sim \frac{Ze^2}{m_e b^2}$$

So $E_{abs} \sim \dfrac{2Z^3 e^6}{3 m_e^2 c^3 b^3 v}$

We expect energy peaked around frequencies $\nu \sim v/b$.

See HKT p. 209-211 for derivation of Bremsstrahlung opacity.

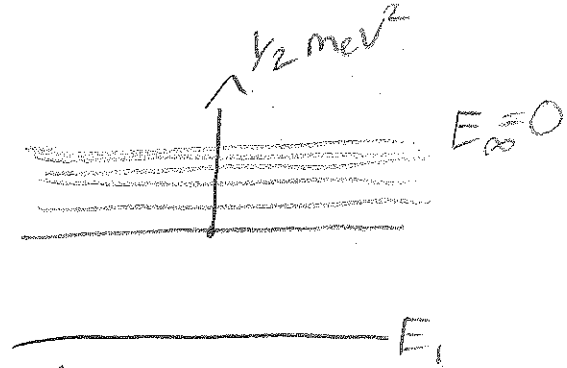
Result:

$$K_{ff} \simeq 4\times10^{-24} \frac{Z^2 n_e n_I T^{-3.5}}{\rho} \; cm^2/g$$

$$\approx 10^{23} Z^2 \left(\frac{\rho}{g/cm^3}\right)\left(\frac{T}{K}\right)^{-3.5} cm^2/g$$

In Sun, $\rho \sim 1$, $Z^2 \sim 1$, so K_{ff} is larger than K_T for $T \lesssim 10^7$ K

<u>Bound-free absorption / Ionization</u>



$$h\nu = \frac{1}{2} m_e v^2 - E_n$$

Only occurs if $h\nu > |E_n|$

Opacity must be computed by summing absorption cross sections for <u>many</u> atomic energy levels. It turns out that an approximate form for realistic plasmas is

$$K_{BF} \approx 4\times10^{25}\, Z \,\rho\, T^{-3.5} \; cm^2/g$$

↳ mass fraction of metals

Note for $Z \sim 10^{-2}$, K_{BF} is comparable to K_{ff}. Although metals usually contribute little mass, their electron energy levels are more complex. They thus have more transitions, higher cross sections, and typically dominate the BF opacity.

Bound-bound opacity is usually less important, except at discrete frequencies of energy level differences, eg, Balmer lines. This opacity causes absorption lines visible in stellar spectra.

It turns out for K_{ff}, K_{bf}, K_{BB}, we have

$$K \propto \rho T^{-3.5}$$

These are called Kramer's opacity, which is dominant form of opacity in interiors of low-mass stars (except where $T \gtrsim 10^7 K$).

In higher mass stars, ρ is typically smaller, T is larger, and electron scattering is most important.

Rosseland mean opacity

K is highly dependent on frequency ν, but we can calculate a transparency-weighted mean opacity

$$K_R^{-1} = \frac{\pi}{acT^3} \int \frac{1}{K_\nu} \frac{\partial B_\nu}{\partial T} d\nu$$

We must weight by transparency ($1/K_\nu$) because photons will leak out through low-opacity holes in frequency space.

To derive this, consider flux at frequency ν per unit frequency,

$$U_\nu = \frac{4\pi}{c} B_\nu$$

$$F_\nu = -D_\nu \nabla U_\nu$$

$$= \frac{-4\pi}{3\rho} \frac{1}{K_\nu} \frac{\partial B_\nu}{\partial T} \frac{dT}{dr}$$

$$\Rightarrow F = \int F_\nu d\nu = \frac{-4\pi}{3\rho} \frac{dT}{dr} \int_0^\infty \frac{1}{K_\nu} \frac{\partial B_\nu}{\partial T} d\nu$$

And defining from radiative diffusion equation,

$$F = -\frac{4acT^3}{3\kappa_R \rho}\frac{dT}{dr}$$

we obtain equation above.

In stellar interiors, κ_R is used, as deviations only affect structure at low optical depth near surface of star.

In stellar atmospheres, κ_ν is used, because radiative transfer at different frequencies is crucial for absorption line formation, which is then used to calculate surface properties (eg, T_{eff}, gravity, metal abundances).