Ay123 Problem Set 3

due Wednesday, November 7, 9:00 am

1. **Equation of state and the Chandrasekhar Mass** (10 points)

   Consider a white dwarf composed of electrically neutral gas, consisting of helium ions and fully degenerate electrons. Assume that the ions contribute negligibly to the pressure and that the gas obeys a polytropic equation of state, $P = K\rho^\gamma$.

   (a) Using the Fermi-Dirac distribution for non-relativistic electrons, derive the relationship between density and pressure, and hence appropriate value of γ and K.

   (b) Using the mass-radius relations we derived for polytropes, derive the mass-radius relation for a white dwarf. Calculate the radius of a $1\,M_\odot$ white dwarf.

   (c) Now assume the helium white dwarf is supported by highly relativistic degeneracy pressure. Use the Fermi-Dirac distribution to derive the appropriate values of γ and K, and then derive its mass. Express the mass in units of $M_\odot$.

   (d) Recalculate this mass for a relativistic white dwarf made of pure gold (^{197}Au). Gold is currently worth 39.47 \$/g. What is the value of this golden dwarf?

   (e) If the golden dwarf has a radius of 3000 km and an internal temperature of 10^7K, will most of its mass be liquid gold or solid gold?

   (f) R-process elements such as gold typically have large opacities at astrophysical conditions. If the surface layers of the golden dwarf have an opacity of $\kappa = 10\,\mathrm{cm}^2/\mathrm{g}$, what is the surface pressure at the photosphere? If the gold at the photosphere can be treated as an ideal gas and has a temperature of 10^4 K, what is its density? The radius of a gold atom is $a_{\mathrm{AU}} = 1.74 \times 10^{-8}$ cm. Is the gold at the surface likely to be pressure ionized?

2. **Nuclides and Kilonova Event Rates** (6 points)

   (a) Consult the table of nuclides. Identify two stable, pure s-process nuclides and two stable, pure r-process nuclides.

   (b) In the Sun, the mass fraction of r-process nuclides is $X_{rp} \sim 10^{-7}$. The stellar mass of the Milky Way is $M_{\mathrm{MW}} \sim 10^{11}\,M_\odot$. Assuming similar abundances in other stars, estimate the total r-process mass within the Milky Way.

   (c) From observations of GW170817, it is estimated that $M_{rp} \sim 0.03\,M_\odot$ of r-process nuclides were expelled from the kilonova. Assuming r-process elements are produced solely in neutron star mergers, estimate the number of neutron star mergers that have occurred in the Milky Way, and the average neutron star merger rate over the $T_{\mathrm{MW}} \sim 10$ Gyr lifetime of the Milky Way.

   (d) Type Ia supernovae synthesize $M_{\mathrm{Fe}} \sim 0.5\,M_\odot$ of iron, whose mass fraction in the Sun is $X_{\mathrm{Fe}} \sim 10^{-3}$. Estimate the average Type Ia supernovae rate of the Milky Way.

   (e) Estimate the number of neutron star mergers that have occurred in a dwarf galaxy with metallicity $\sim 10^{-1}$ that of the Milky Way, and stellar mass $M_{\mathrm{gal}} \sim 10^6\,M_\odot$. Do we expect all dwarf galaxies to be enriched in r-process elements such as Europium?

3. **Free Electrons at Low Temperatures** (5 points)

Consider a stellar photosphere made of pure hydrogen.

(a) Use the Saha equation to calculate the fraction of free electrons n_e/n_{tot} for a pure hydrogen gas. Plot your result as function of temperature. Assume $n_{tot} = 10^{17}$ cm^{-3} as is appropriate for the solar photosphere, where n_{tot} is the total number of nuclei (ionized + neutral).

In a more realistic situation, both hydrogen and other elements contribute to the number of free electrons n_e that shows up in the Saha equation. Let's consider a simple model in which we have a gas of hydrogen and a single metal. The metal has an ionization energy $\chi_m \approx 5$ eV (as is appropriate for sodium). For simplicity, we will assume that the metal can only be ionized once (this is reasonable at the low temperatures where the presence of the metal is important). Assume that $n_{tot,m} \approx 10^{-6} n_{tot,H}$ (again reasonable for sodium) and set the product of g's for the metal to 1.

(b) Calculate the fraction of free electrons $n_e/n_{tot,H}$ as a function of temperature for the combined hydrogen + metal gas. As in a), assume $n_{tot,H} = 10^{17}$ cm^{-3}. Make a plot showing the ratio of n_e for part b) to n_e for part a) as a function of temperature. Below what temperature does the assumption of a pure hydrogen gas start to become inaccurate for predicting n_e? This contribution of metals to free electrons is critical for understanding many aspects of cool stellar photospheres, the ISM, accretion disks, etc.

NOTE: Solving part b) will require a numerical root finding technique, which you can solve using Python, Mathematica, etc.

4. **On-screen plots with MESA** (4 points)

Carry out the pgstar Lab by Frank Timmes at

http://cococubed.asu.edu/mesa_summer_school_2018/agenda.html

which can be downloaded under the schedule for Monday. Ope the pdf file (mesa_ss_2018) and follow the instructions.

(a) Make plots as instructed, similar to those on slides 16, 20, 24, and 32. It will be easiest to use the *.png files generated by MESA.