Ay123 Problem Set 1

Due on Wednesday October 10, 9:00 am

1. **The scale of the Sun** (10 points)

   Measuring the scale of astronomical objects is not easy. In this problem, you may use relevant physical laws and constants (e.g., G and σ_B), but do not look up measurements of the Sun or solar system.

   (a) The angular radius of the Sun is 16 arcmin, and the radiant flux received at the top of the Earth's atmosphere (the "solar constant") is 1.388×10^6 erg s^{-1} cm^{-2}. Using these data alone, calculate the effective temperature of the Sun.

   (b) Venus has an orbital period of 225 days. Using Kepler's laws, what is its semi-major axis in units of AU?

   (c) At conjunction with the Sun, astronomers on Earth emit a radio pulse directed toward Venus. It takes 276 seconds to detected the radio waves that reflect off Venus. Assuming circular orbits for the Earth and Venus, use this information to compute the distance in 1 AU.

   (d) Use your results above to compute the absolute mass, radius, and luminosity of the Sun in cgs units.

2. **Stars are gases** (10 points)

   The average density of the Sun is slightly larger than that of water.

   (a) Provide a quantitative relation between the temperature and density of a star which indicates when we can treat it as a gas throughout its interior, in spite of the very high densities. Is our assumption valid at the center of the Sun, where the density is about 100 times the average density? Note that your answer should only involve classical physics, no quantum mechanics.

   (b) If all stars have roughly the same central temperature (that of the sun), use a scaling argument to determine the stellar mass at the which the simple non-interacting ideal gas assumption breaks down.

3. **A toy star** (15 points)

   Assume that a star obeys a linear density model:

   $$\rho(r) = \rho_c \left(1 - \frac{r}{R}\right),$$

   where ρ_c is the central density, R is the radius of the star, and $a \geq 1$.

   (a) Find an expression for the central density in terms of R and the total mass M.

   (b) Use the equation of hydrostatic equilibrium and zero boundary conditions to find the pressure as a function of radius. Your answer will be in the form $P(r) = P_c \times f(r/R)$, where $f(x)$ is a function that you will determine. What is the dependence of the central pressure P_c in terms of M and R?

   (c) What is the central temperature T_c assuming the ideal gas equation of state? How does it scale with mean particle mass?

(d) Find the ratio of the radiation pressure to the gas pressure at the center of this star as a function of the total stellar mass, expressed in units of $M_\odot$. At what mass does radiation pressure become comparable to the ideal gas pressure?

(e) Write down an explicit expression for the total gravitational potential energy of this toy star, and verify that the virial theorem is exactly satisfied.

4. **Stellar coronae** (5 points)

The Sun's corona is extremely hot and diffuse, extending far beyond the photospheric radius of the Sun. Because it is transparent and its mass is neligible, its structure is different than the sub-photospheric layers.

(a) Consider a spherically symmetric stellar corona and assume that the temperature in the corona is constant and that the mass of the corona is negligible compared to the mass of the star M. Solve hydrostatic equilibrium for the density profile as a function of radius given a density ρ_b at a base radius $r_b \simeq R$ (the solar radius).

(b) What is the pressure in the corona as $r \to \infty$? Note that it is finite! Assume further that the star is surrounded by a low density and low pressure interstellar medium (ISM), such that the ISM pressure is much less than asymptotic coronal pressure inferred here. Comment on the implications for the *dynamics* of the stellar corona.

5. **Using the MESA stellar evolution code** (10 points)

Go to http://mesa.sourceforge.net/, and read about how to download and install MESA. You will likely want to run this on a Mac or Linux machine. I highly recommend first installing the MESA SDK, as instructed on the Mesa website. Download and install the latest public release of MESA, version 10398 as of September 30, 2018.

(a) Run the default stellar model located in mesa/star/work/. At time step 10, what is the core temperature and surface temperature of the model?

(b) Evolve a $1\,M_\odot$ model up the red giant branch.

-Change the starting mass to $1\,M_\odot$.

-You may want to disable the `create_pre_main_sequence_model` option.

-Make sure to change the stopping condition so that the model will evolve beyond core hydrogen depletion.

-Edit the plotting bounds of the HR diagram in the `inlist_pgstar` file so that you can see the model evolve on an HR diagram.

-Evolve the star until its luminosity exceeds $100\,L_\odot$.

-Using the `history.data` located in the `LOGS` folder, make an HR diagram for your stellar evolution track. Also plot $\log L$ as a function of time. You may use whatever program you want (e.g., Python, IDL, etc.). `mesa_reader` is a useful too for analyzing MESA output, and can be downloaded at
https://github.com/wmwolf/py_mesa_reader/
with documentation at
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