Stellar Atmospheres

⇒ Learning Outcomes:

By the end of this lecture, students will be able to:

1) Recognize + define the components of the radiative transfer equation
2) Explain how optical depth is related to observable features of stellar atmospheres
3) Summarize how stellar spectra can be used to classify stars + measure different stellar properties
4) Identify at least one area of current research involving stellar atmospheres

⇒ Outline:

I. What is a stellar atmosphere?

II. Review of radiative transfer concepts

III. What can stellar spectra tell us?

- Components of stellar spectra
- What quantities can we measure from these components?

IV. Open questions / research areas

I. What is a stellar atmosphere?

- Definition: transition region from dense + opaque stellar interior → interstellar medium
- usually divided into parts: photosphere *
  chromosphere } outer layers
  corona

  * we'll focus on the photosphere in this class, since this is where most of the observable features are produced!

- Features of the photosphere:
  (slide 2 – temp + density profiles of solar atmosphere)
  - temp decreases the farther out you go from the star
  - relatively thin + dense (compared to chromosphere/corona)
  - most of the photons are emitted from this layer
    ↳ what exactly does this mean? let's review radiative transfer...

II. Review of radiative transfer concepts

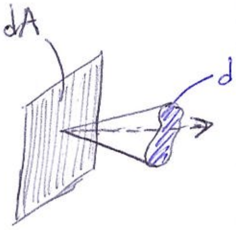
⇒ The basic idea behind radiative transfer:

"light can either be emitted or attenuated"
↳ (absorbed, scattered, etc.)

Let's describe this concept more quantitatively...

- Definition: specific intensity $I_\nu = \dfrac{dE}{dA\,dt\,d\nu\,d\Omega}$ [1]

= energy per area per time per frequency per solid angle
↑
carried by rays of light



dA = "light source" (ex., star)

$d\Omega$ = "region where your detector lives" (ex., eye, telescope mirror)

II. (cont.)

In free space, I_ν doesn't change along a ray: $\frac{dI_\nu}{\underset{\substack{\uparrow \\ (s\,=\,\text{length along ray})}}{ds}} = 0$ [2]

... but space isn't really empty. Light can be absorbed* or emitted — we describe these processes using the radiative transfer equation (RTE):

$$[3]\quad \boxed{\frac{dI_\nu}{ds} = -\underset{\swarrow}{\alpha_\nu} I_\nu + j_\nu} \longrightarrow \text{emission coefficient}$$

absorption coefficient

$$\alpha_\nu = \underset{\text{(opacity)}}{\kappa_\nu}\,\underset{\text{(mass density)}}{\rho} = \overset{\text{(\# density)}}{n}\,\sigma_\nu \leftarrow \text{(absorption cross-section)}$$

$$j_\nu = \frac{dE_{\text{emitted}}}{\underset{\substack{\uparrow \\ \text{volume}}}{dV}\,d\Omega\,dt}$$

*(side note: This gets gross when scattering is involved, because you end up needing to integrate all the emission scattered from $d\Omega'$ into $d\Omega$ — this leads to an integrodifferential equation that must be solved numerically. We'll ignore scattering for now.)

The RTE has an even nicer form if we write it in terms of a unitless quantity called optical depth (τ_ν): (slide 3 – physical intuition for τ)

[4] $d\tau_\nu = \alpha_\nu\, ds$ (can integrate along length of ray to get τ_ν)

With this quantity, the RTE becomes:

$$[5]\quad \boxed{\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu}$$

$\hookrightarrow$ where S_ν is the "source function": $S_\nu = \frac{j_\nu}{\alpha_\nu}$ [6]

$\Rightarrow$ Let's consider a simple solution to the RTE:

$$(*)\quad I_\nu(\tau_\nu) = \underbrace{f(\tau_\nu)}_{\text{some function of } \tau_\nu} e^{-\tau_\nu}$$

II. (cont.)

Plug this solution (*) into the RTE (Eq. 5):

$$-I_\nu + e^{-\tau_\nu} \frac{df}{d\tau_\nu} = -I_\nu + S_\nu$$

Solve for $f(\tau_\nu)$: $f = \int_0^{\tau_\nu} S_\nu e^{t_\nu} dt_\nu + c_0$

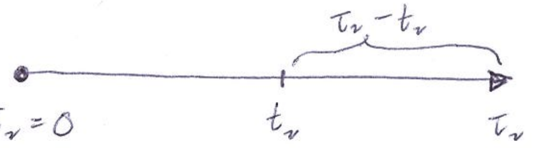
And solve for I_ν : $I_\nu(\tau_\nu) = e^{-\tau_\nu} \int_0^{S_\nu} S_\nu e^{t_\nu} dt_\nu + c_0 e^{-\tau_\nu}$

⇓

(plug in $\tau_\nu = 0$ to get $c_0 = I_\nu(0)$)

⇒ Final solution :

$$I_\nu(\tau_\nu) = \int_0^{\tau_\nu} \underbrace{S_\nu(t_\nu)}_{\left(\substack{\text{newly generated}\\ \text{intensity}}\right)} \underbrace{e^{-(\tau_\nu - t_\nu)}}_{\left(\substack{\text{attenuation}\\ \text{of } S_\nu}\right)} dt_\nu + \underbrace{I_\nu(0) e^{-\tau_\nu}}_{\left(\substack{\text{attenuation of}\\ \text{original intensity}}\right)} \qquad [7]$$



→ Notes about this solution (Eq. 7):

① Eq. 7 represents radiative transfer along one line of sight — but when we actually observe stellar photospheres, we see a sum over many lines of sight

② To fully solve Eq. 7, need to know the source function S_ν !

↓

Physically, these 2 things can tell us a lot about the observable properties of stars! Note that the star becomes opaque at $\tau \sim 1$, so this sets the observable properties. Then ① + ② can explain some of the features we see:

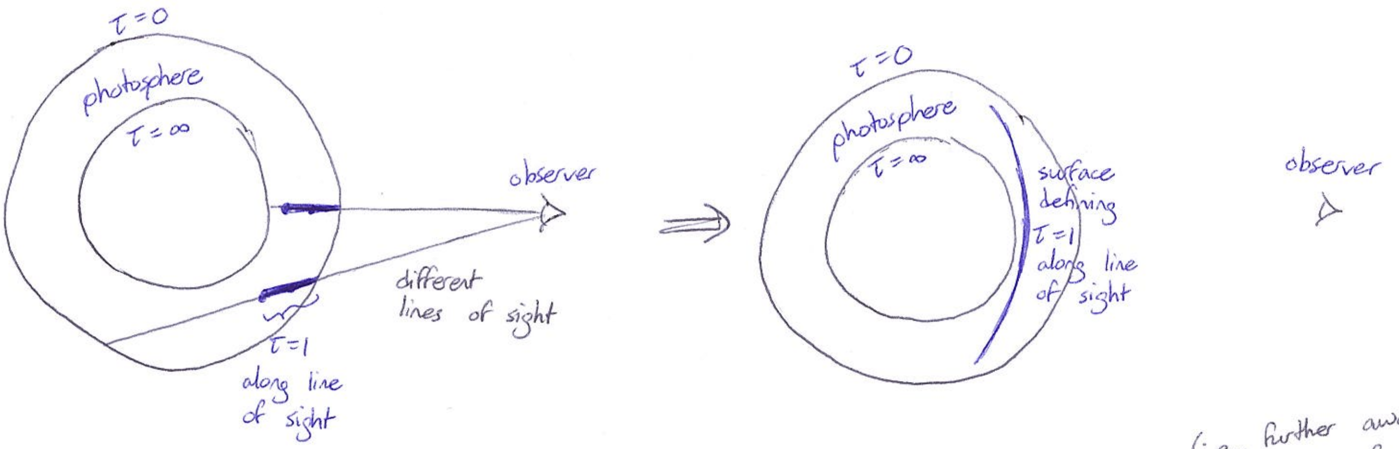
① Limb darkening comes from observing the star through different lines of sight

(Slide 4 — example of solar limb darkening)

↳ the "limb" (edge of the stellar disk) looks darker than the center of the star!

II. (cont.)

① Limb darkening (cont.)



(i.e., further away from center of star,

At the limb of the star, you're looking at a shallower depth into the photosphere
⇒ the photosphere is cooler further away from the center of the star, + therefore darker here!

Also, if you average over the entire surface where $\tau = 1$ along the line of sight (the blue curve in the right-hand figure above), you end up with $\tau = \frac{2}{3}$ when measured radially from the center of the star

② Most of the light from a star comes from a blackbody source function

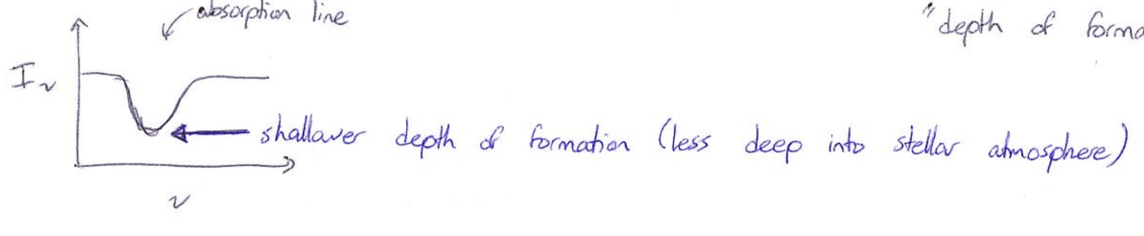
$$S_\nu(\tau) \approx B_\nu(T) \leftarrow B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} \quad [8]$$

As we just saw, ~~we~~ the star becomes ~opaque at $\tau = \frac{2}{3}$ on average, so we define the effective temperature to be the temp at $\tau = \frac{2}{3}$:

$$T_{\text{eff}} = T(\tau = \tfrac{2}{3})$$

③ Note that τ isn't constant across frequencies! This produces absorption lines (Slide 5 – think-pair-share in-class activity)

More opaque frequencies/wavelengths = looking at shallower $\underbrace{\text{depth into the photosphere}}_{\text{"depth of formation"}}$!



III. What can stellar spectra tell us?

Now that we understand how radiative transfer mathematically describes the observable properties of stars through τ_ν, let's understand what physically sets τ_ν...

- Components of stellar spectra
  (slides 6 + 7 – examples of stellar spectra + their components)
  - continuum:
    - similar to blackbody spectrum with $T = T_{eff}$
    - continuum emission = thermal
      continuous opacity = bound-free, free-free, e^- scattering
  - spectral lines:
    - line emission = e^- de-excitation
      line opacity = e^- excitation (bound-bound)
      } energy level diagram (slide 8 – example diagrams)
- To $\underbrace{\text{make spectral lines}}$, need to know 2 things:
  for ex., to predict τ_ν for, say, a hydrogen line (slide 9 – hydrogen energy level diagram)

① How much of any given element is there?
  ex – to make a hydrogen line, need hydrogen!
  - hence, spectra can tell us about chemical abundances – we'll get to this later

② How are electrons distributed in different energy levels (+ therefore available to move to other levels)?
  ex – to make the Hα line (hydrogen line where electrons move from $n=2 \rightarrow n=3$ states), need electrons to be in $n=2$ state!
  - to answer this question, need 2 equations:

1) Boltzmann equation: how many e^- in each energy level?

$$\frac{N_b}{N_a} = \frac{g_b}{g_a} e^{-(E_b - E_a)/kT} \qquad [9]$$

\# densities of atoms in each level

degeneracies

energies at each level

- this is valid if: - atoms are mostly excited by collisions with other atoms
  - thermal equilibrium (atom KEs are distributed as Maxwell-Boltzman

III. (cont.)

2) <u>Saha equation</u>: how many atoms are ionized?

$$\frac{N_{i+1}}{N_i} = \frac{2}{n_e} \frac{Z_{i+1}}{Z_i} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{-\chi_i / kT} \quad [10]$$

N_{i+1}/N_i: # densities of atoms in each ionized state

n_e: # density of e^- (depends on e^- pressure)

χ_i: ionization energy

- need to know <u>partition function</u> Z = weighted avg. of neutral atoms in each energy state

$$Z = g_1 + \sum_{j=2}^{\infty} g_j \, e^{-(E_j - E_1)/kT} \quad [11]$$

~~together, the~~

⇒ together, the Boltzmann + Saha equations determine the temperature (+ pressure) at which a given line will be strongest (ex: for Hα line, need electrons to be in $n=2$ state of hydrogen — Eqs. 9 + 10 can be solved to determine the temp at which [illegible] the number of H atoms with e^- in the $n=2$ state is highest!)

↳ (see Problem Set 4, #4)

- What quantities can we actually measure from stellar spectra?
  - <u>temperature</u>
    - from blackbody shape of continuum
    - from temperature sensitive lines

    } (again, see Problem Set 4, #4)
  - <u>surface gravity</u>
    - electron number density (n_e) depends on electron pressure, which is related to surface gravity ($g = G\frac{M}{R^2}$):

$$n_e \propto \rho \propto P \propto g$$

(from ideal gas law $P = \frac{\rho k T}{\mu m_p}$)

(since $P = \frac{2}{3}\frac{g}{\bar{\kappa}}$ at photosphere — recall HW 3)

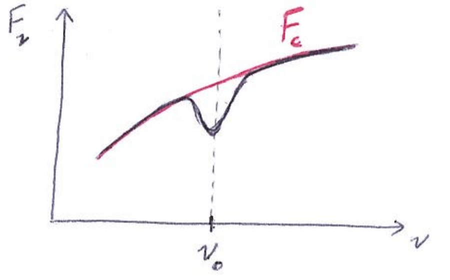
  - <u>chemical abundances</u>
    - from spectral lines — usually by fitting to models

in order to do this, we need to relate the shapes of spectral lines to chemical abundances — that is, we need to define what it means for a line to be "strong"

(slide 10)

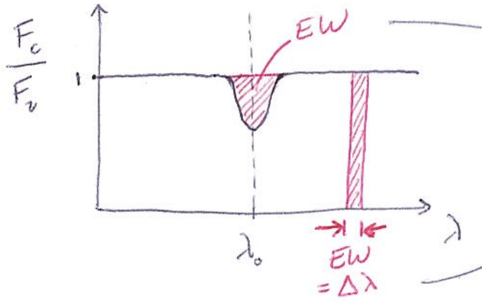
III. (cont.)

Let's consider the flux profile of an absorption line:



F_c is the continuum – the flux assuming the absorption line doesn't exist

Let's normalize the spectrum by the continuum:



The equivalent width (EW) is the area between the continuum + the absorption line.

An equivalent (pun intended) definition: EW is the width of a rectangle with height = 1 that has the same area

Equivalent width is traditionally defined as:

$$W_\lambda = EW = \int \frac{F_c - F_\lambda}{F_c} \, d\lambda \quad [12]$$

⇒ this gives us a number that quantitatively describes the shape of the line!

But how do we describe the shape of the line in more detail? What sets the EW of a line?

⇒ EW depends on F_λ, which depends on the λ-dependent opacity:

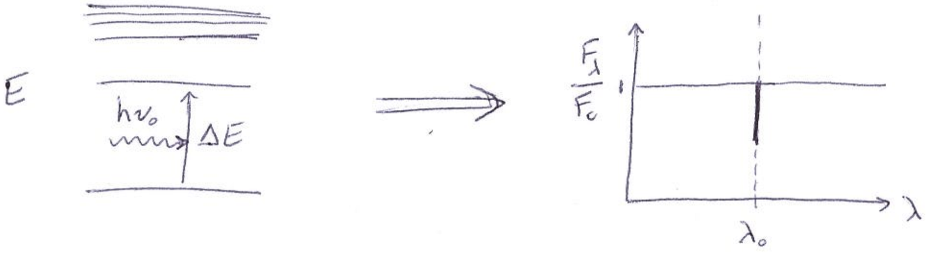
$$F_\lambda \propto S_\lambda(\tau_\lambda)$$

$$\uparrow$$

$$d\tau_\lambda = \int \boxed{\alpha_\lambda} \, ds$$

The opacity (+ absorption coefficient) is determined by atomic/quantum physics! This therefore determines the shape of spectral lines.

- In a single-atom system, spectral lines are infinitely narrow:



single photon produces single energy transition

III. (cont.)

- If all atoms behaved exactly the same, spectral lines would just be Dirac delta functions. But differences in atoms broaden absorption lines:

(1) natural broadening: caused by the Heisenberg uncertainty principle

$$\underset{\substack{\nearrow \\ \text{line width}}}{\Delta E}\ \underset{\substack{\uparrow \\ \text{lifetime of electron in energy level}}}{\Delta t} \gtrsim \frac{\hbar}{2} \qquad [13]$$

- uncertainty in the energy state of the electron ⇒ uncertainty in transition λ
  ⇒ this produces a Lorentzian line profile:

$$L(\lambda) = \frac{A}{1 + \left(\frac{\lambda_0 - \lambda}{w/2}\right)^2} \qquad [14]$$

where λ_0 = central wavelength of line, A = amplitude of profile, and w = "full-width half maximum" (FWHM)

(2) Doppler broadening: caused by Doppler shift of moving atoms

$$\Delta\nu = \nu_0 \frac{v}{c} \leftarrow \text{atom velocity}$$

- if atom velocities follow Maxwellian distribution, this is equivalent to a Gaussian distribution in the radial velocity direction ⇒ get Gaussian line profile:

$$G(\lambda) = A e^{-(\lambda - \lambda_0)^2/(2\sigma^2)} \qquad [15]$$

where λ_0 = central wavelength, A = amplitude, and σ = standard deviation (note: $\text{FWHM} = 2\sqrt{2\ln(2)}\,\sigma$)

(3) collisional (or pressure) broadening: caused by collisions between atoms

- atoms knock e^- out of energy levels by colliding, producing shorter Δt in Eq. 13 ⇒ larger ΔE
  ⇒ like natural broadening, this produces a Lorentzian

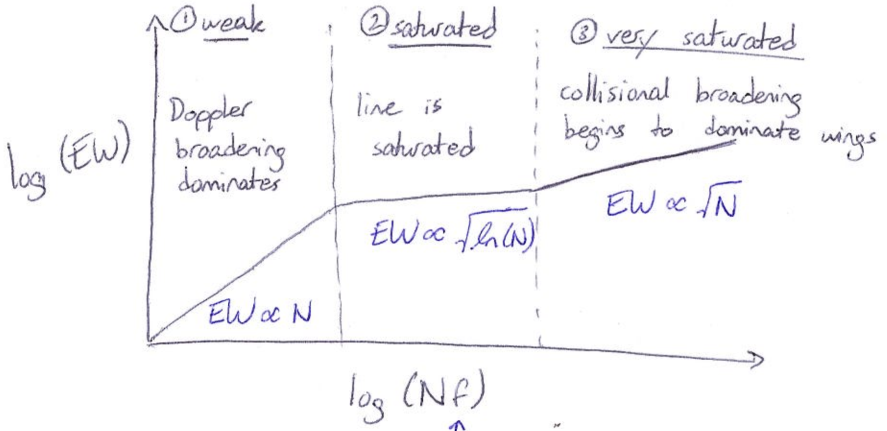
(Slide 11 – Gaussian + Lorentzian line profiles)

- the full line profile is a combination of Gaussian + Lorentzian profiles, called a Voigt function

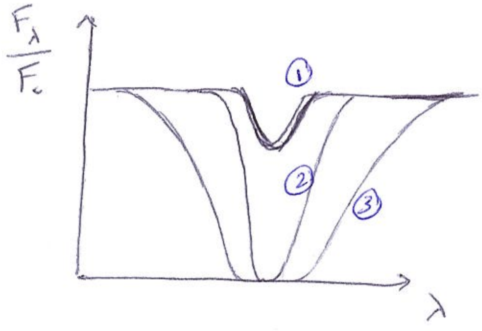
III. (cont.)

- Now that we understand the fundamental physics behind the shapes of spectral lines, let's relate them to physical properties. One way to do this is with something called the curve of growth – the relationship between EW and N
  ↖ # of absorbing atoms

⟹ in-class activity investigating the curve of growth (see attached worksheet + slide 12)
(slide 13 – compare to actual curve of growth)



f = "oscillator strength" – varies from line to line, depends on quantum probability of transition



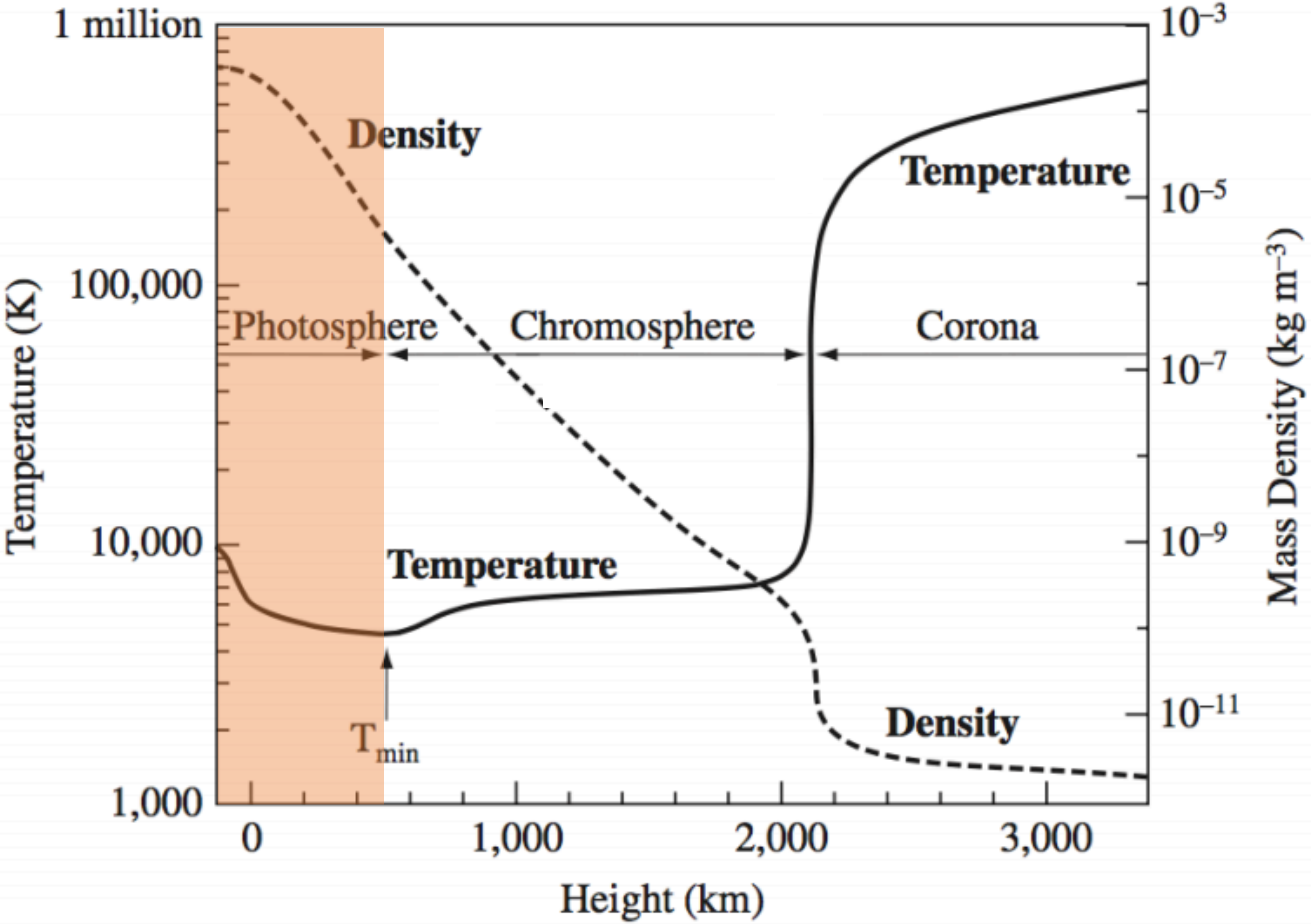
- How to use the curve of growth?
  1) measure EW of line, look up oscillator strength f
  2) use curve of growth to get N (# of atoms in right state to absorb photons)
  3) use Boltzmann + Saha eqns to get fraction of atoms in right state to absorb photons
  4) compute total density of atoms

IV. Open questions + research areas

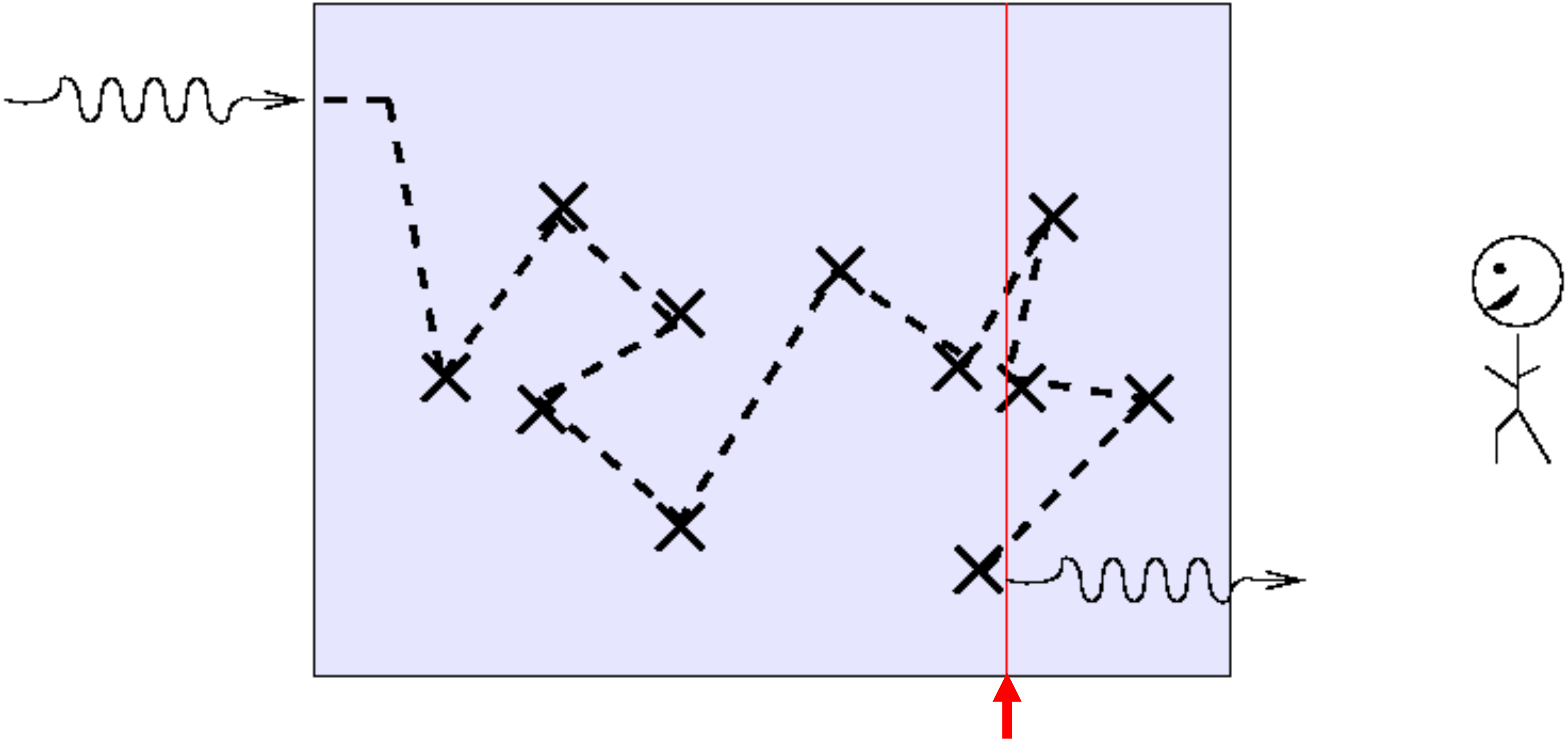
- Stellar activity + magnetic fields (slide 14)
- Using stellar spectra to study galaxies (slide 15 + 16)
- Using stellar spectra to study exoplanets (slide 17)

Stellar Atmospheres

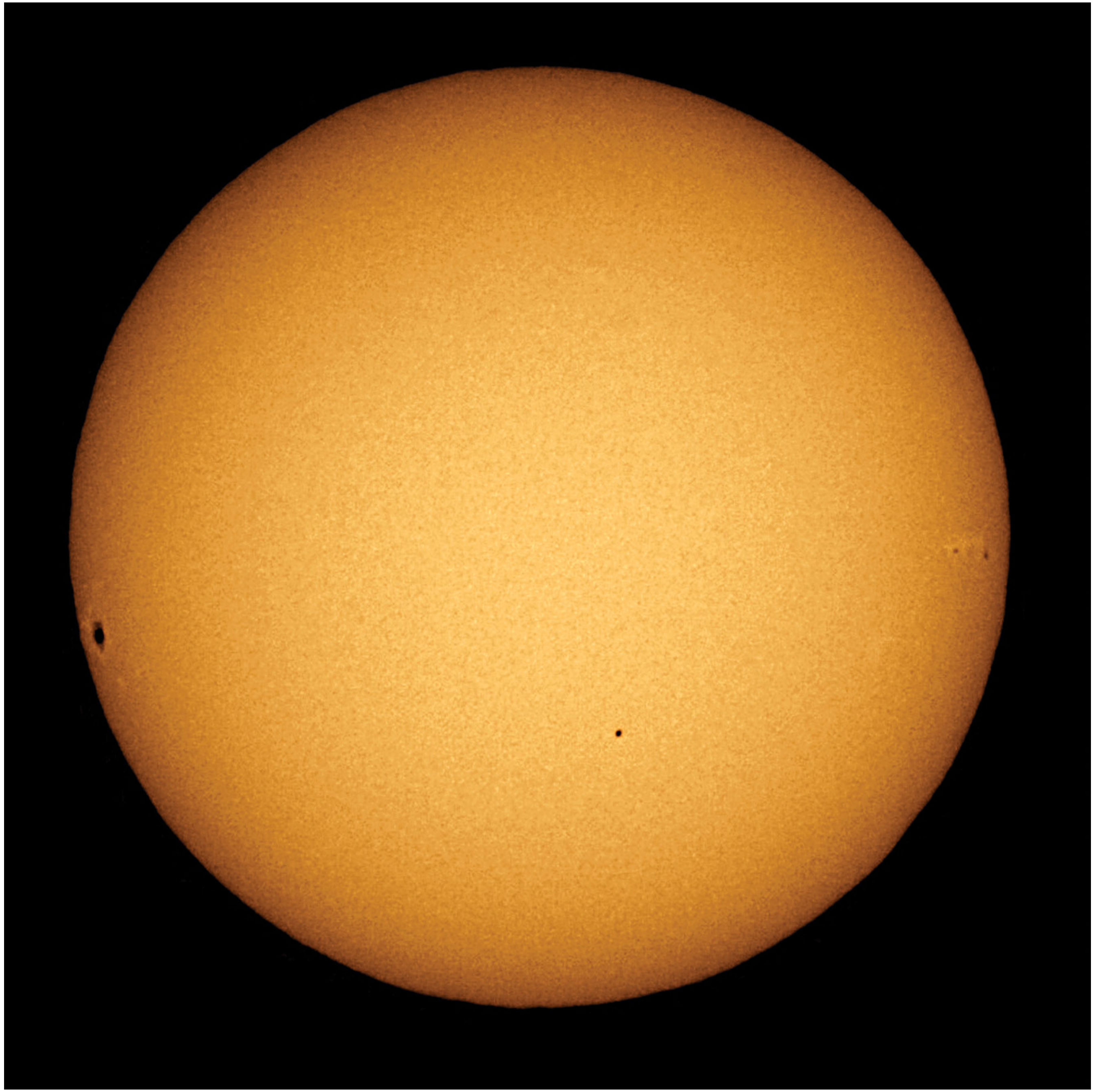
Ay101


1 million
100,000
10,000
1,000
Temperature (K)
10−3
10−5
10−7
10−9
10−11
Mass Density (kg m−3)
0
1,000
2,000
3,000
Height (km)
Density
Temperature
Photosphere
Chromosphere
Corona
Temperature
T_{min}
Density

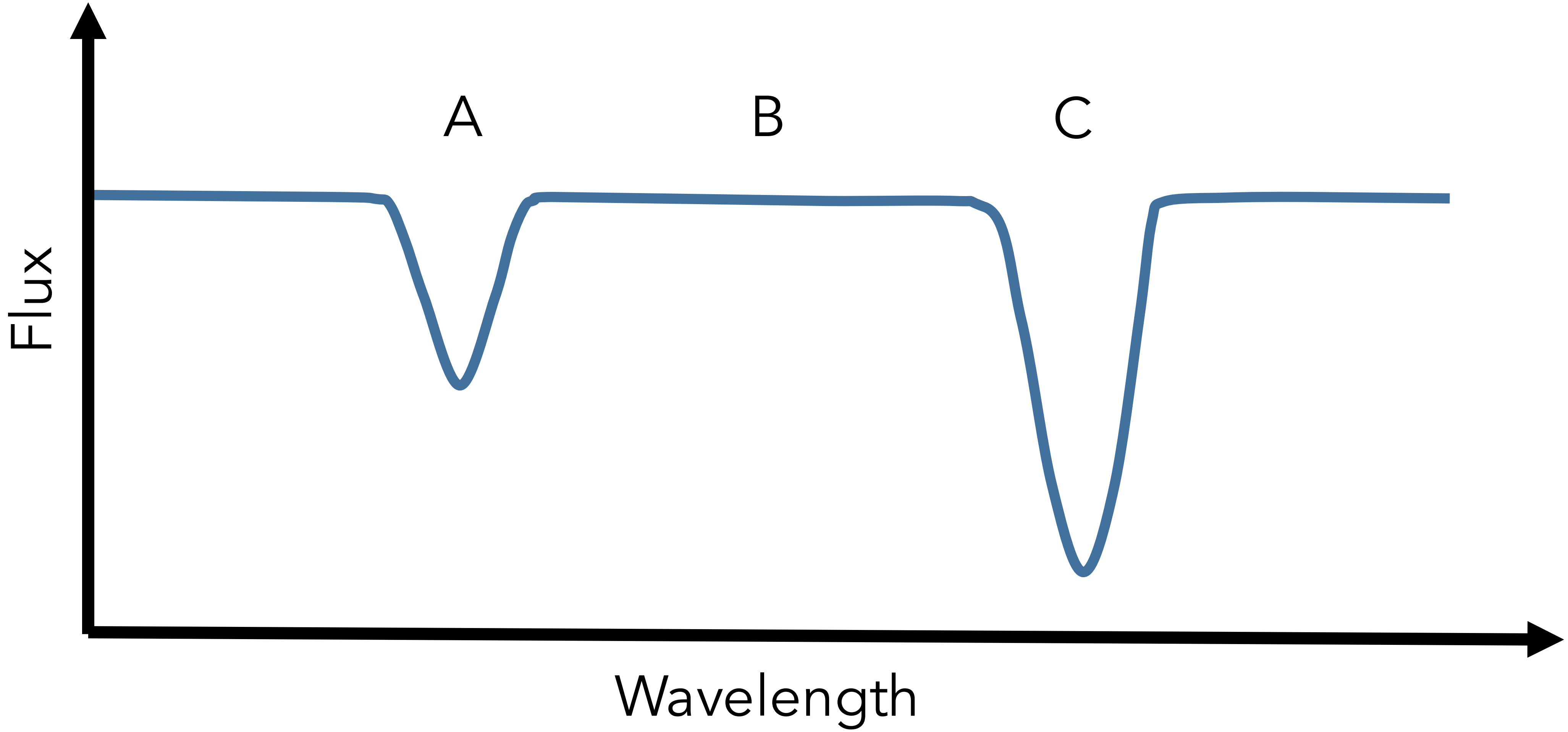
How deep can we see into a slab of material?

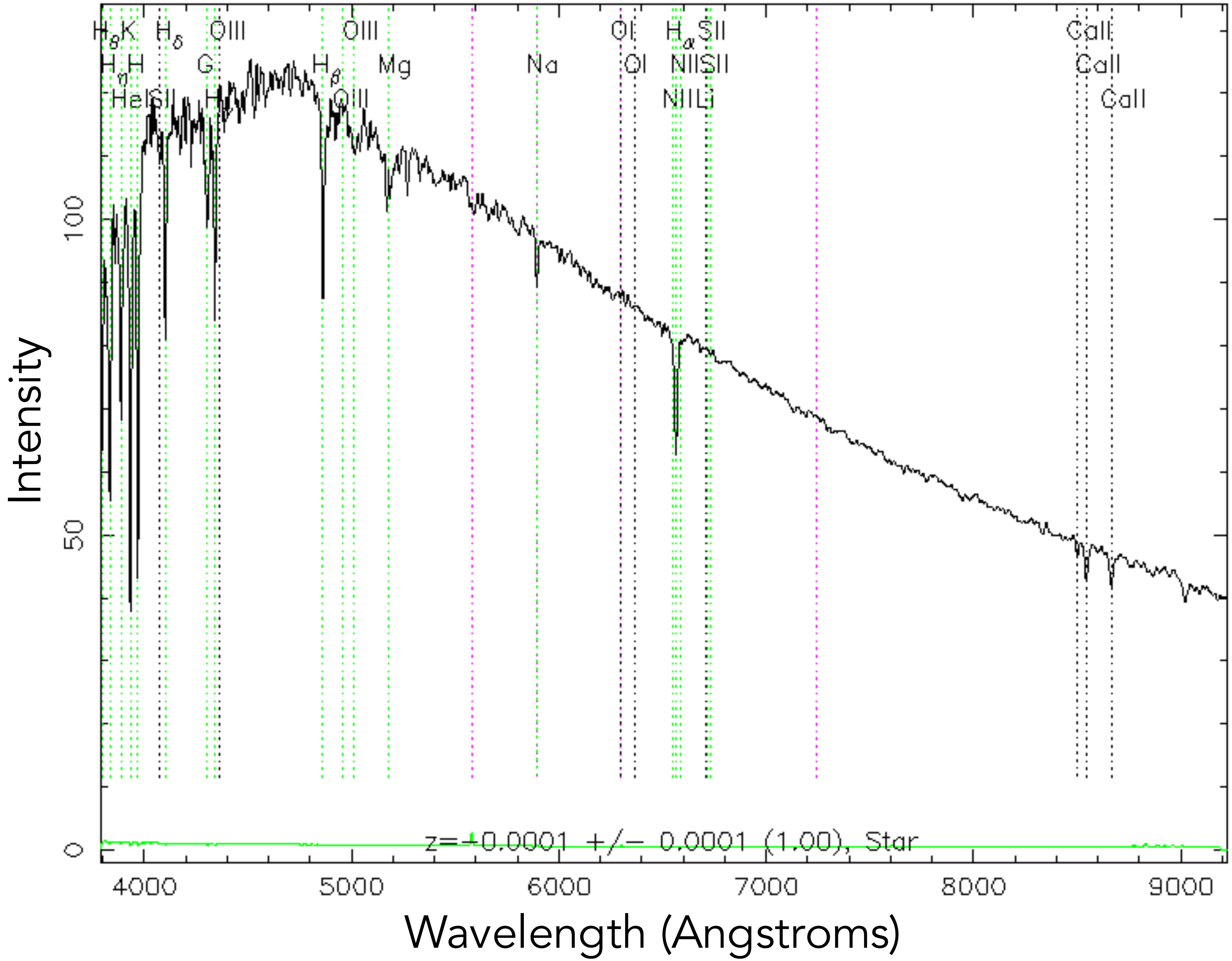


About 1 optical depth **τ**

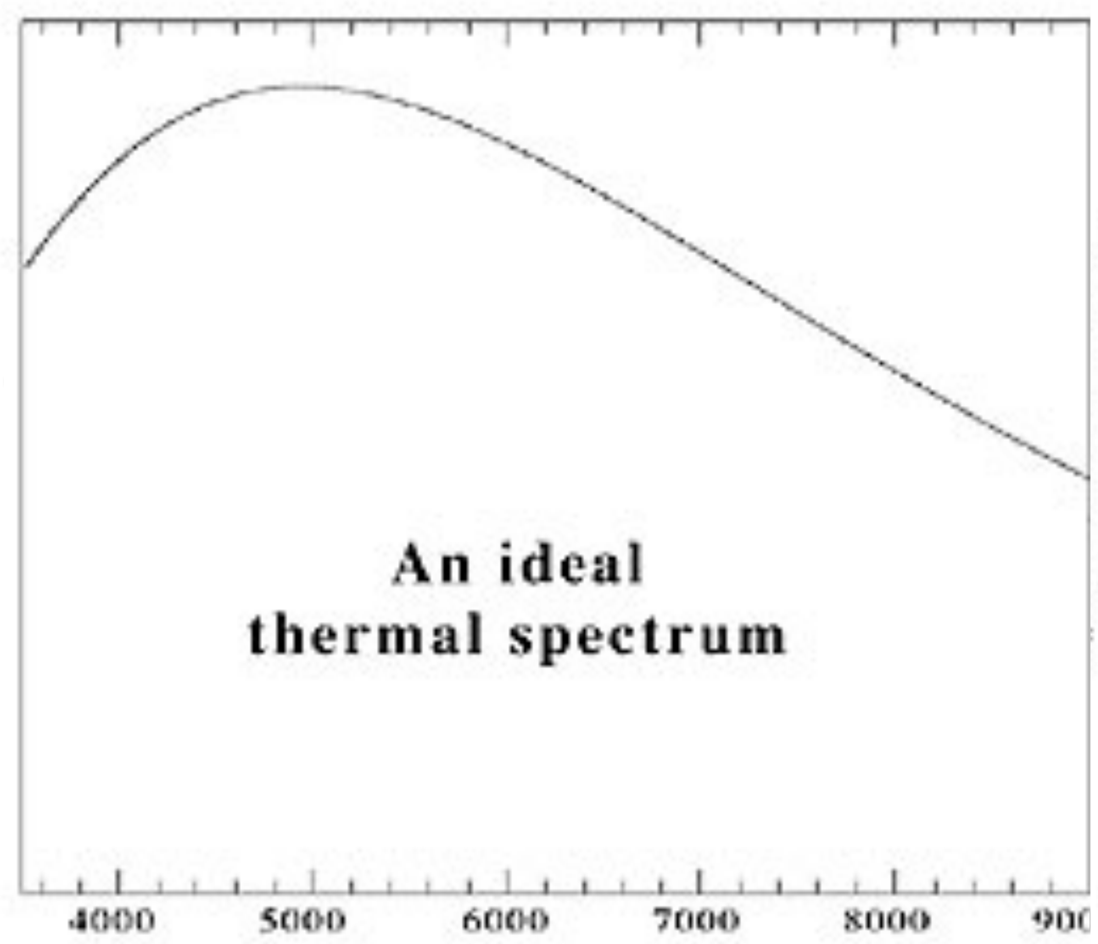


At which wavelength are you seeing deepest into the photosphere?

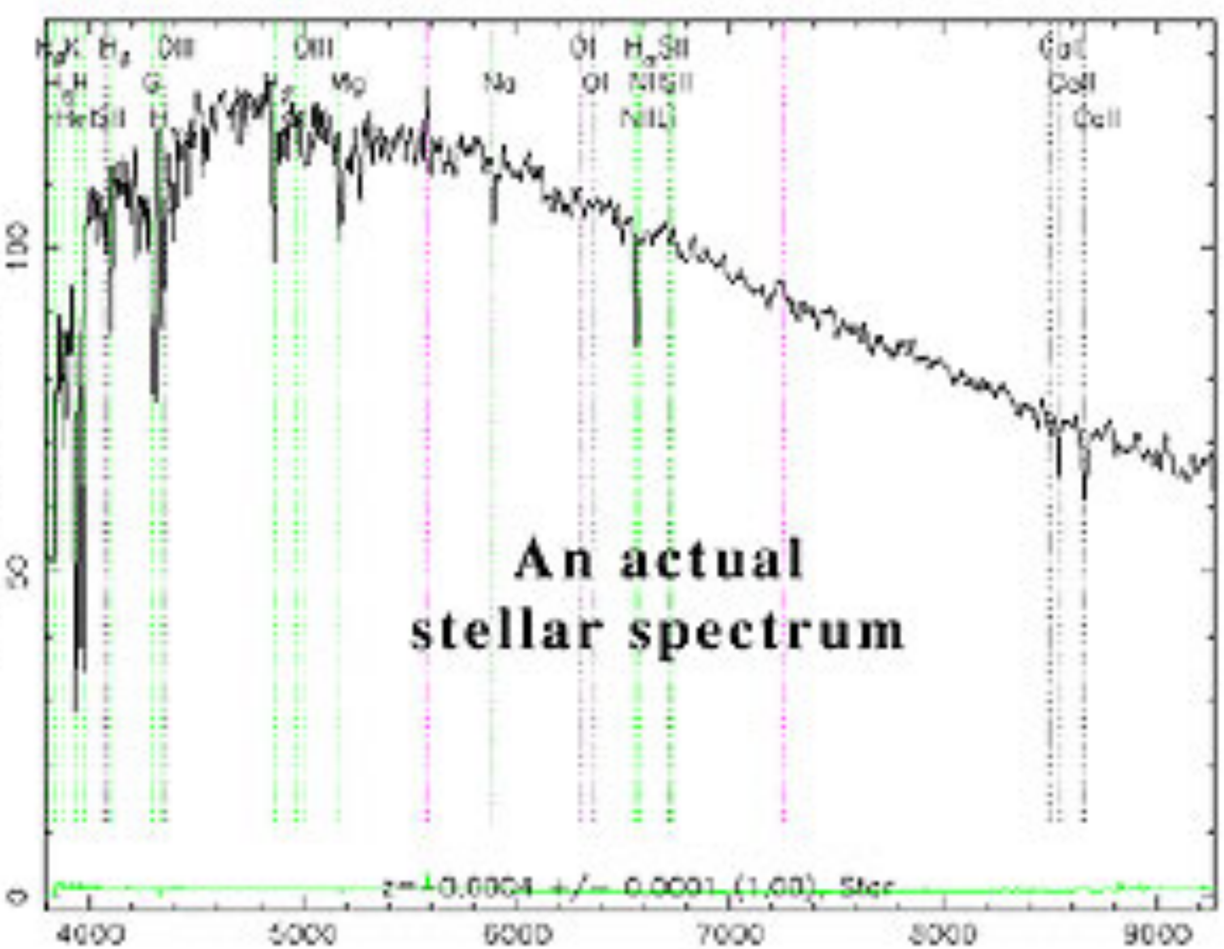



H_ϵ K
H_δ
OIII
OIII
OI
H_α SII
H_η H
G
H_β
Mg
Na
OI
NII SII
HeI SII
H
OIII
NII Li
CaII
CaII
CaII
Intensity
100
50
0
z=-0.0001 +/- 0.0001 (1.00), Star
4000
5000
6000
7000
8000
9000
Wavelength (Angstroms)

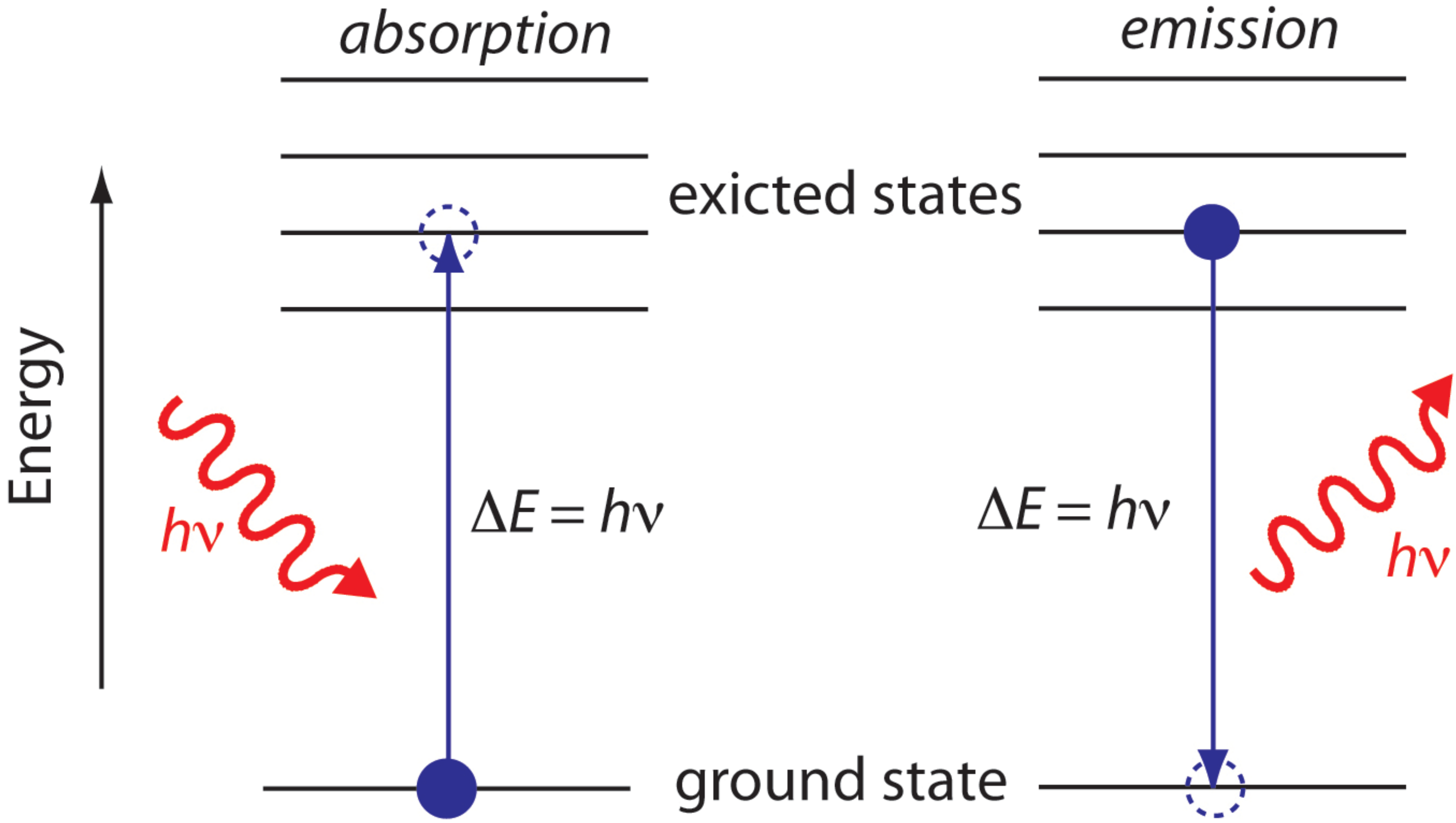
Intensity



Wavelength (Angstroms)



Wavelength (Angstroms)



absorption

emission

exicted states

Energy

hν

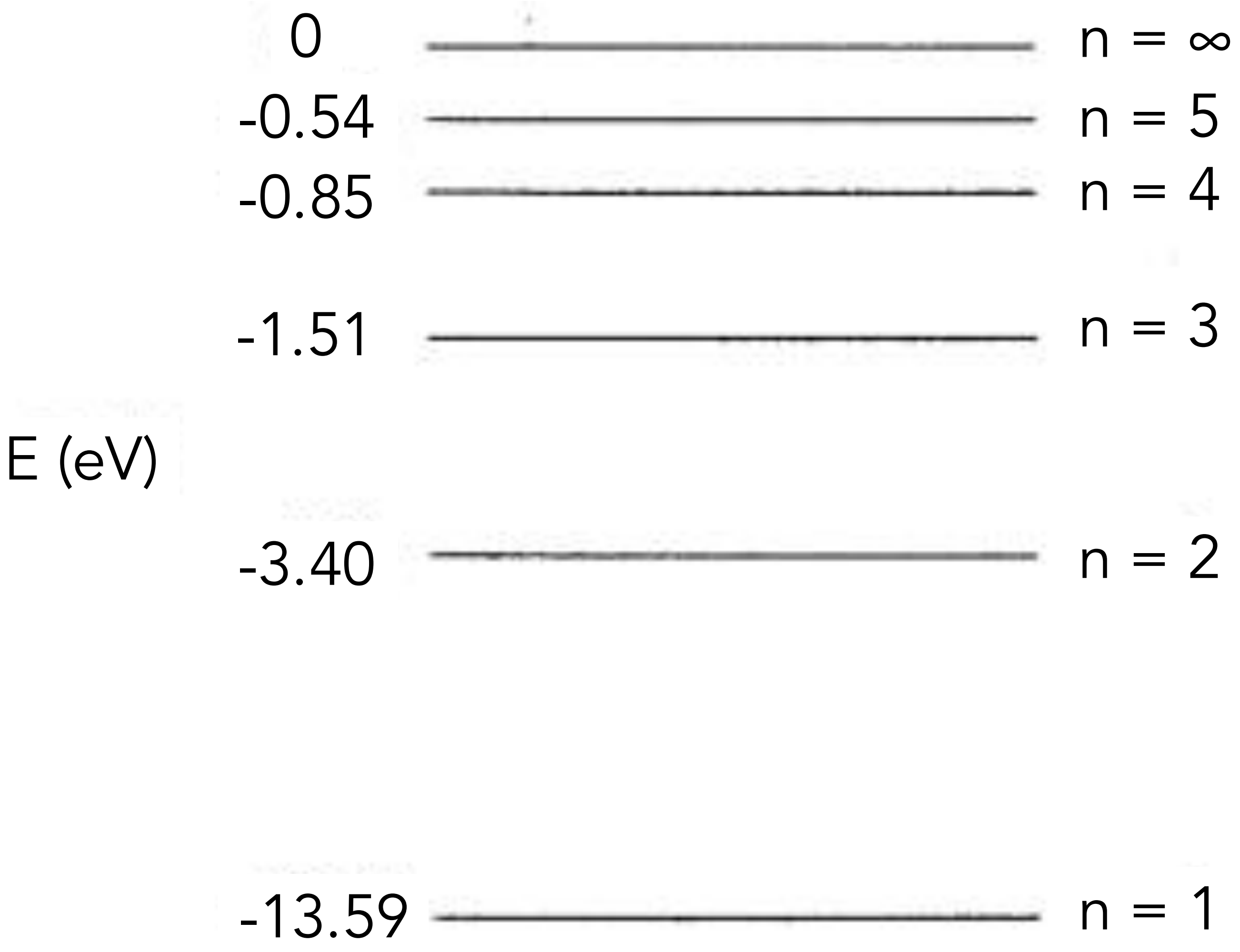
$\Delta E = h\nu$

$\Delta E = h\nu$

hν

ground state

Hydrogen energy level diagram



E (eV)

0 $n = \infty$

-0.54 $n = 5$

-0.85 $n = 4$

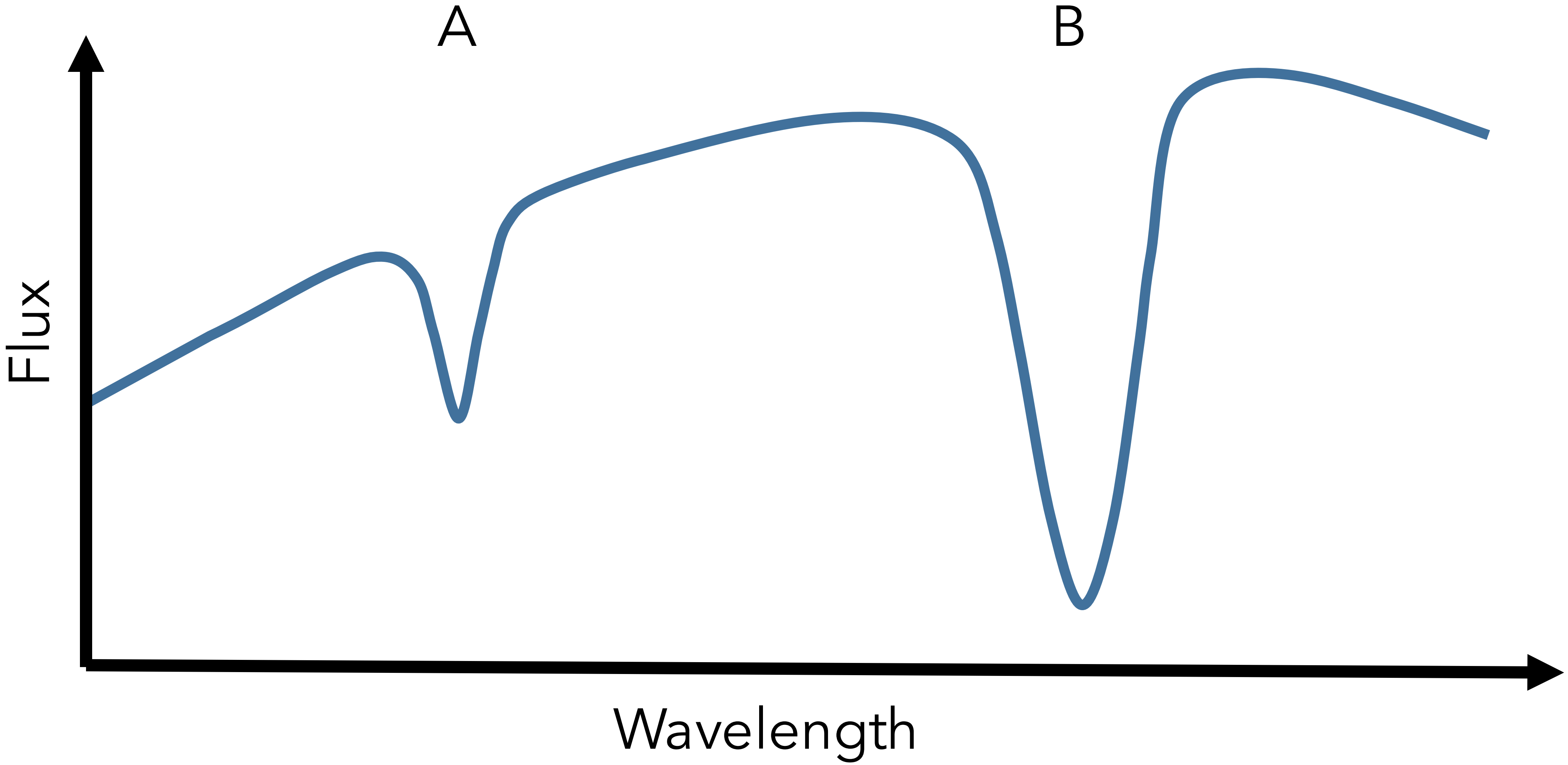
-1.51 $n = 3$

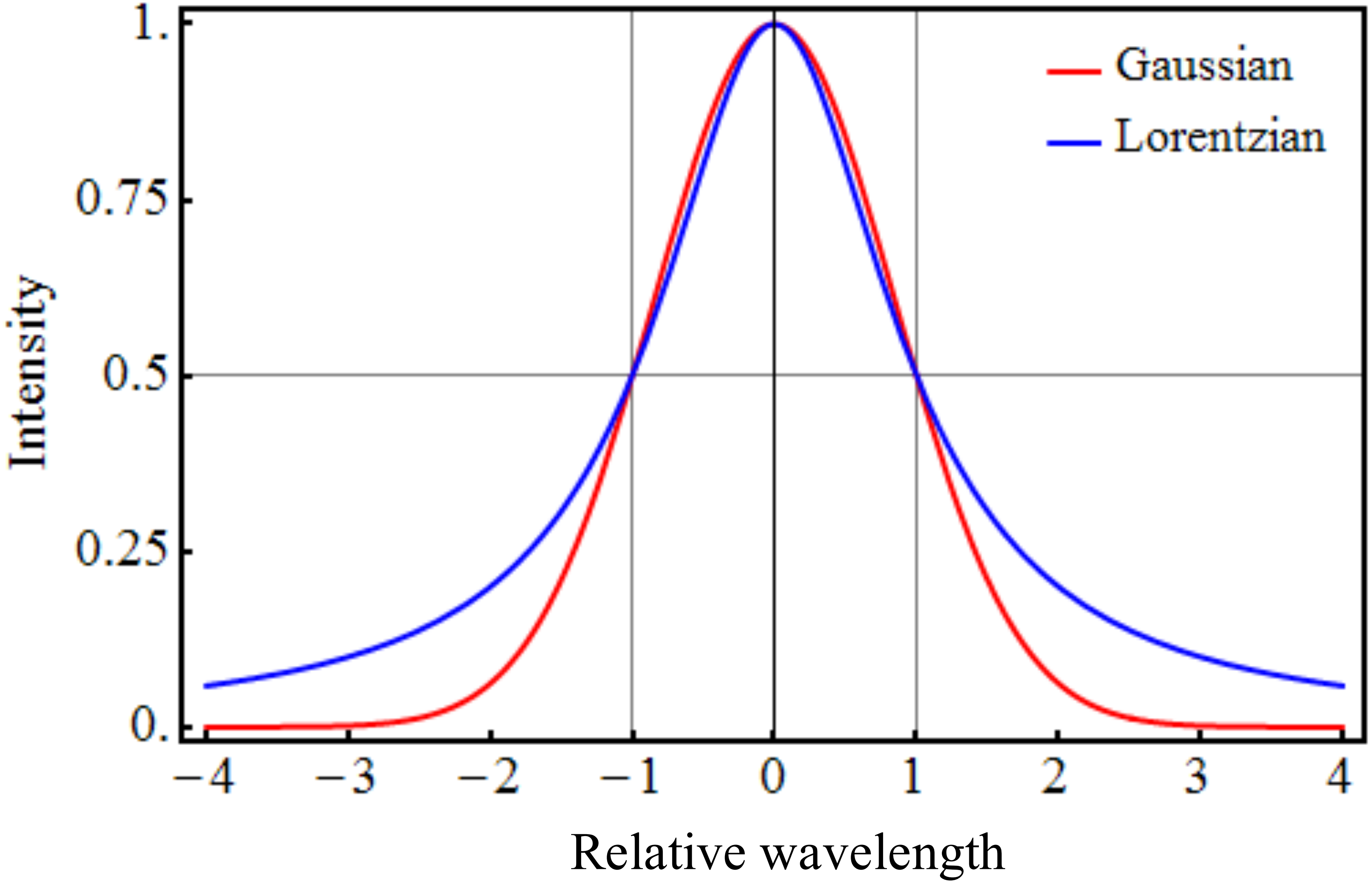
-3.40 $n = 2$

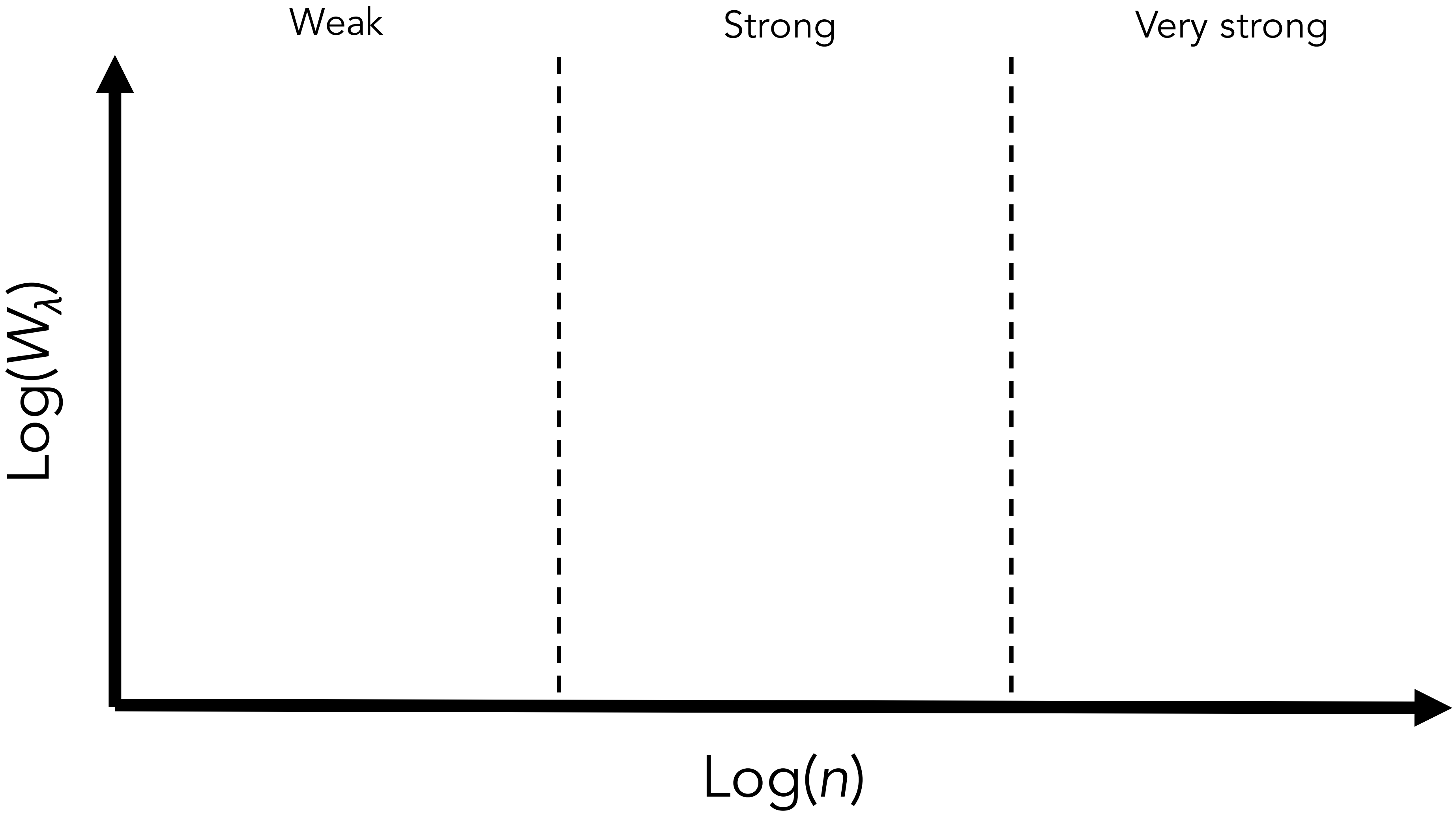
-13.59 $n = 1$

$$\boxed{E = -\frac{13.6\text{ eV}}{n^2}}$$

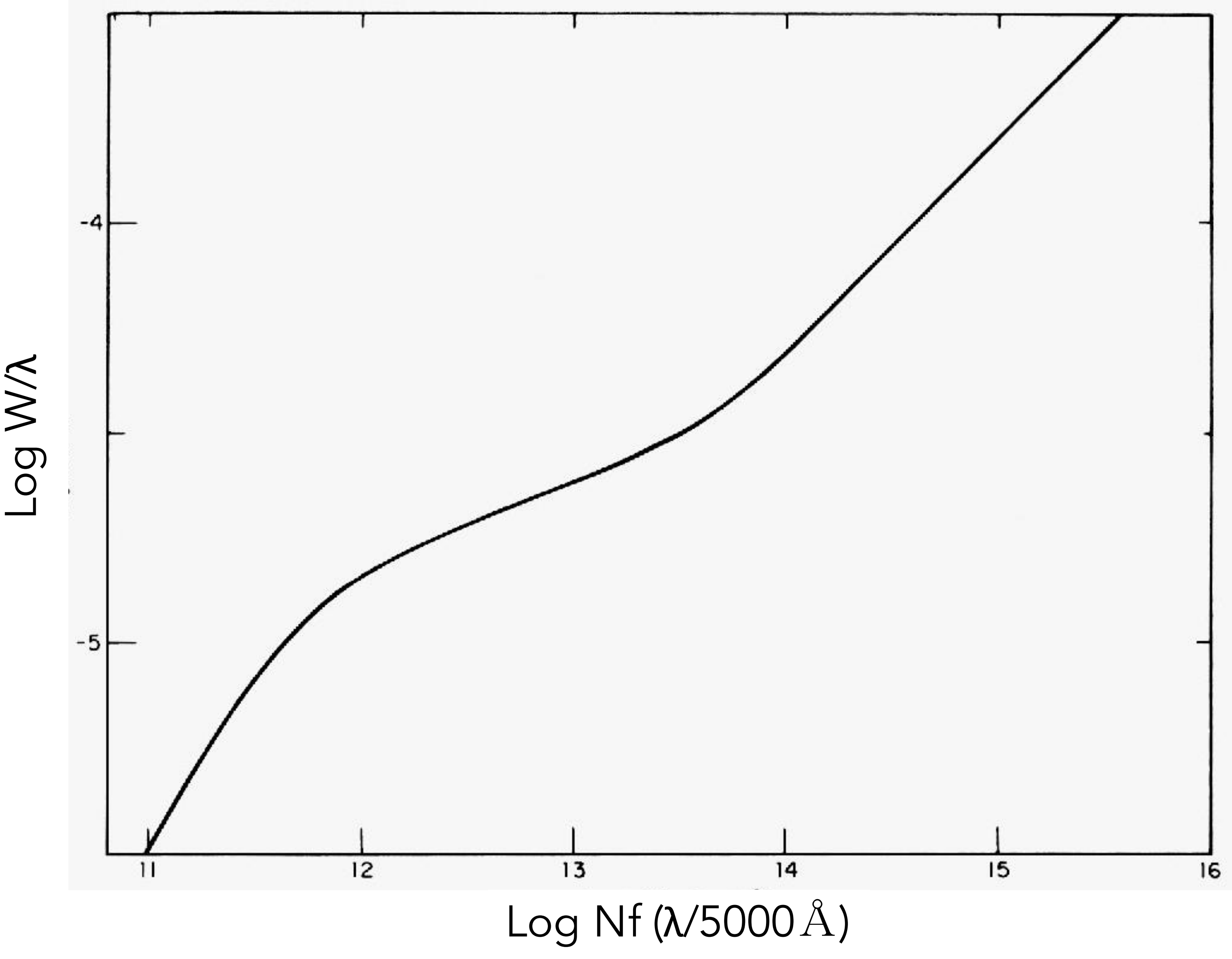
Which line is stronger?



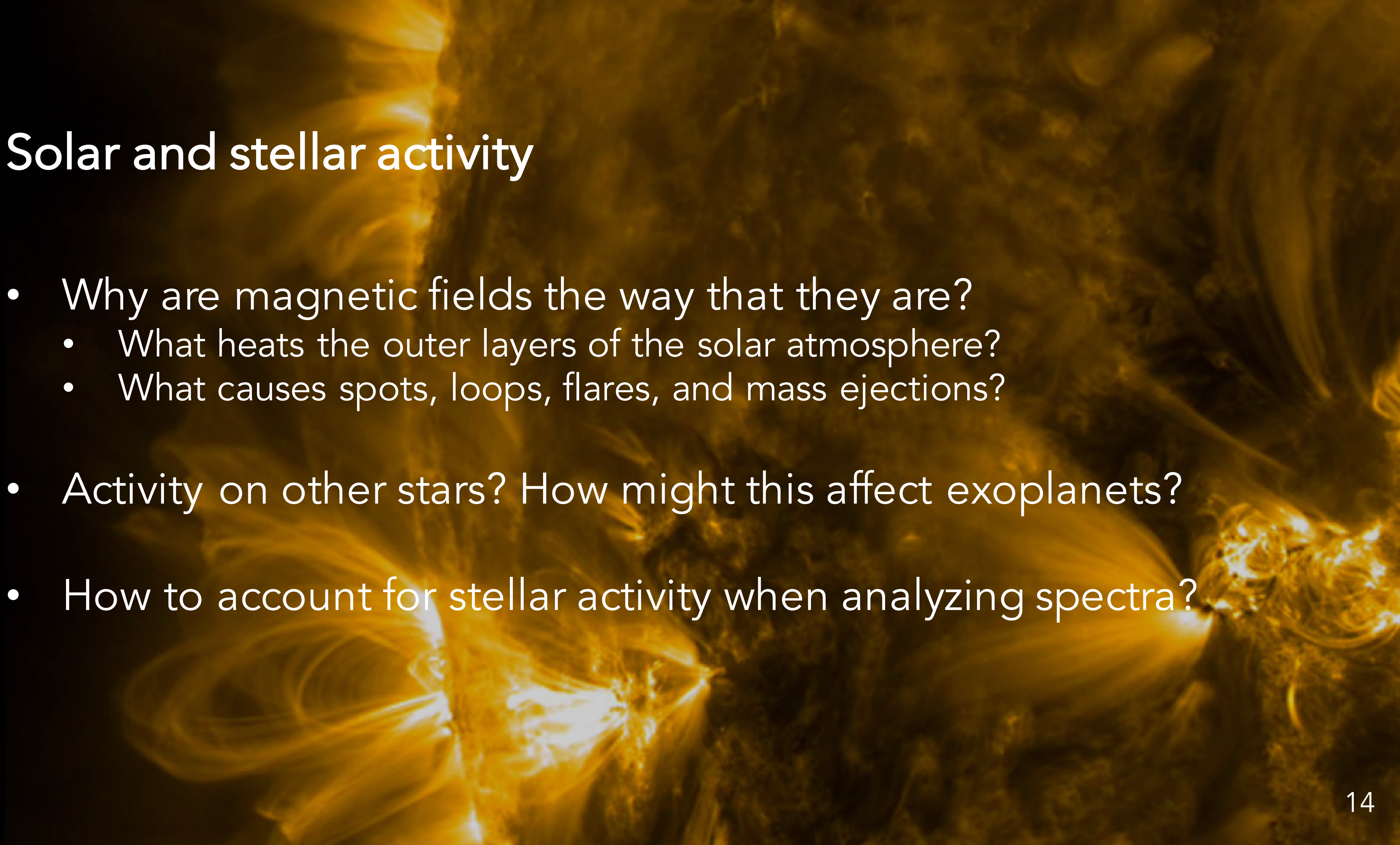

Gaussian
Lorentzian
Intensity
1.
0.75
0.5
0.25
0.
−4
−3
−2
−1
0
1
2
3
4
Relative wavelength


Weak
Strong
Very strong
$\mathrm{Log}(W_\lambda)$
$\mathrm{Log}(n)$

A general curve of growth for the Sun



Solar and stellar activity

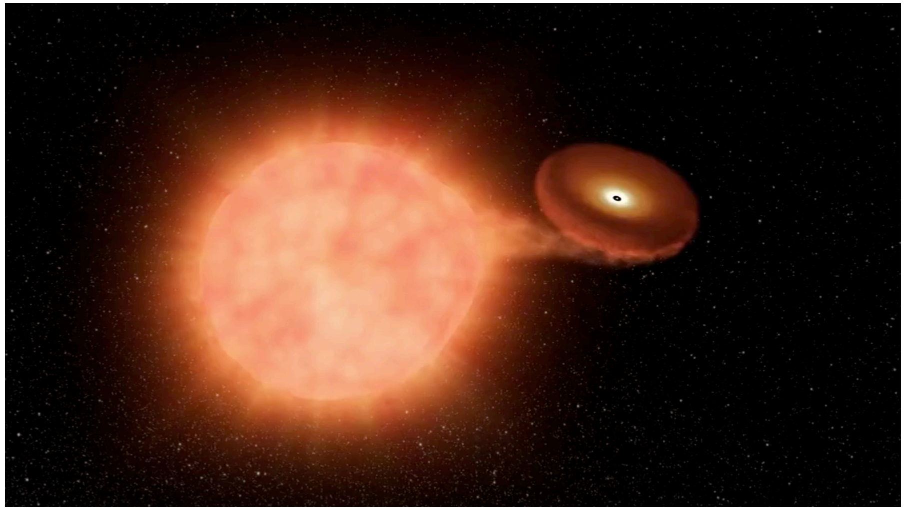


- Why are magnetic fields the way that they are?
  - What heats the outer layers of the solar atmosphere?
  - What causes spots, loops, flares, and mass ejections?
- Activity on other stars? How might this affect exoplanets?
- How to account for stellar activity when analyzing spectra?

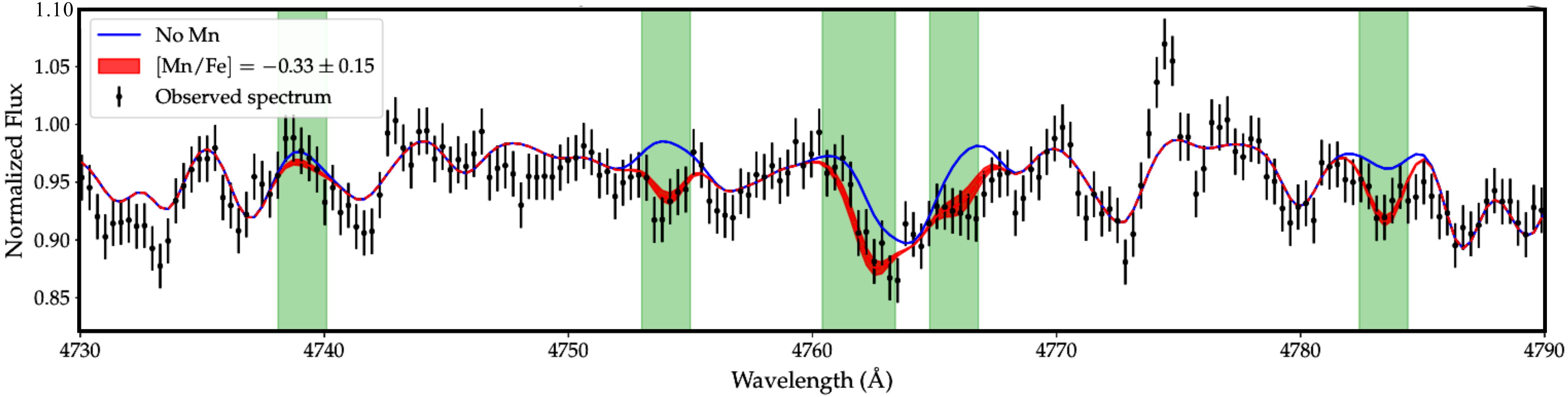
Using stellar spectra to study galaxies

- The search for the first stars
- Stellar abundance patterns in galaxies
- Galactic archaeology
  - Untangling the Milky Way’s and Andromeda’s formation history
  - Tracing “nucleosynthetic” events (events that make elements)

My research: studying the physics of supernovae



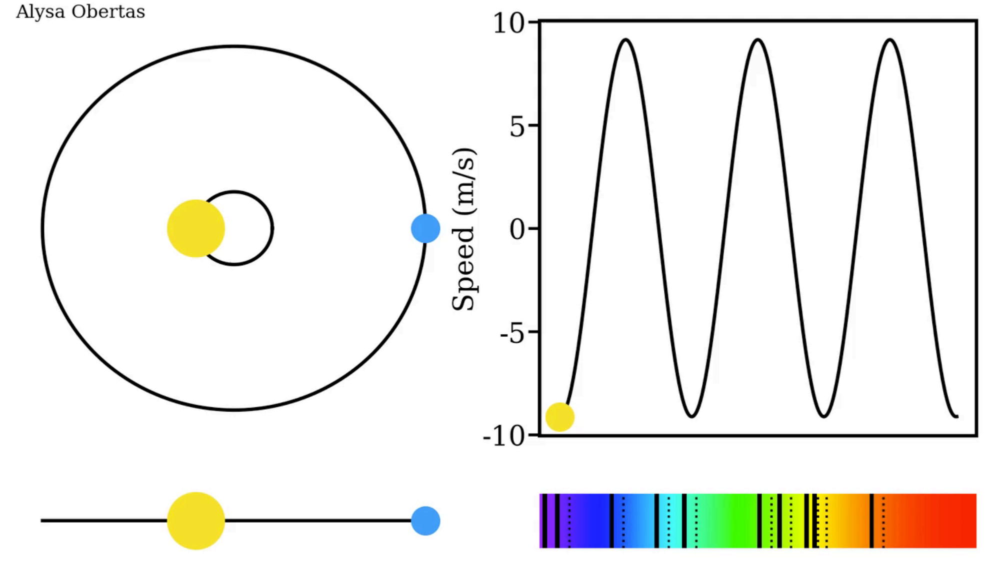
vs.



de los Reyes et al. (submitted)

Using stellar spectra to study exoplanets

- Chemical abundances in planet atmospheres
- Finding exoplanets using the radial velocity method



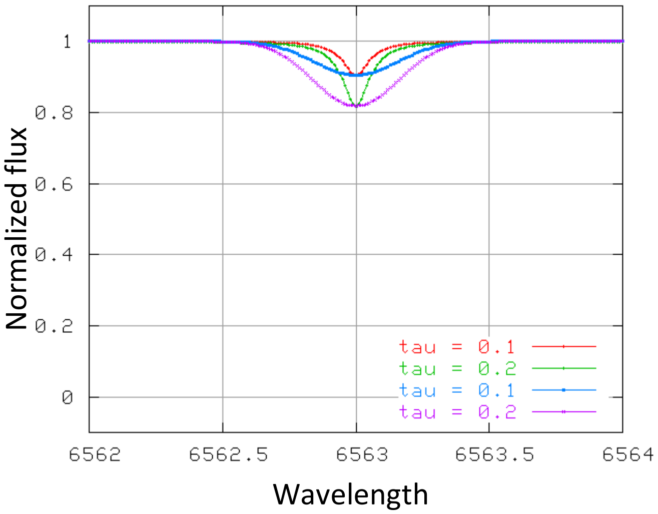
Ay101: The Curve of Growth

The **curve of growth** is the relationship between an absorption line's equivalent width W_λ and the number density of absorbing atoms n. In this activity, we'll investigate the parts of this curve.

1) Using the definition of optical depth τ, how does τ scale with number density n?

$$\tau = \int \alpha\, ds = \int \sigma\, n\, ds, \textit{ so assuming density is constant with } s\textit{: } \boldsymbol{\tau \propto n}$$

2) The figure below shows Gaussian (blue and purple) and Lorentzian (red and green) line profiles for a weak line ($\tau << 1$). Which type of line profile dominates?

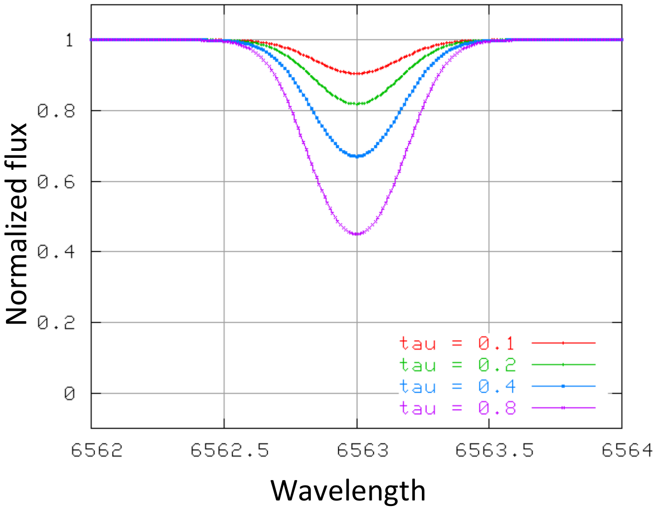


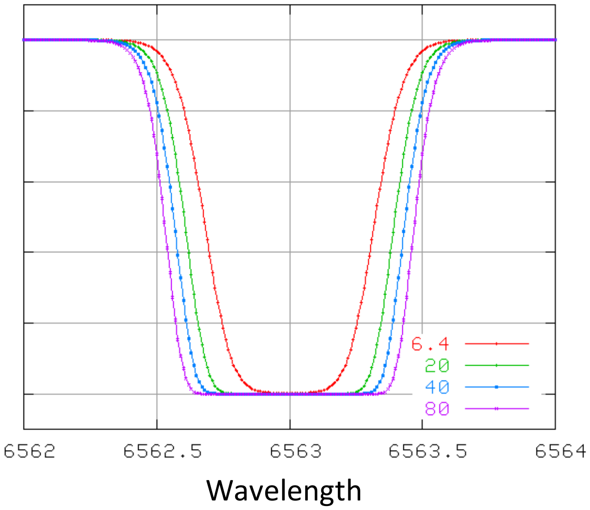
In a Voigt line profile (a combination of Lorentzian and Gaussian profiles), ***the Gaussian profile dominates*** *for weak lines.*

3) For the line profile you answered in #2, the left plot below shows weak lines ($\tau << 1$) while the right plot shows strong lines ($\tau > 1$).

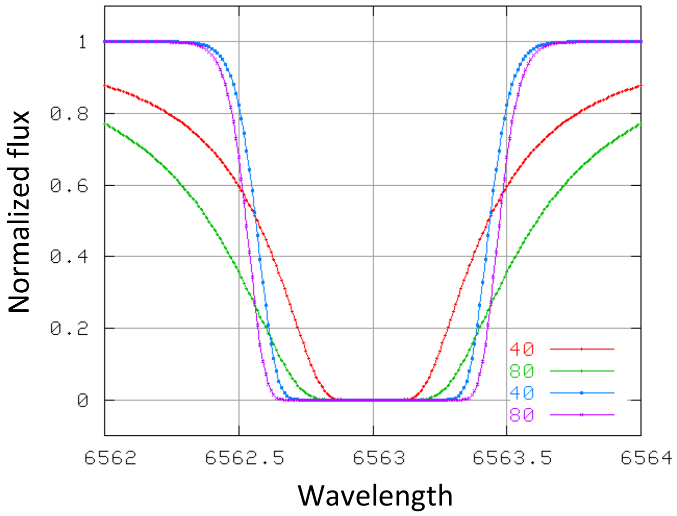
   a. In both plots, does W_λ increase or decrease as you increase τ? **increase**

   b. In which plot does W_λ depend *less strongly* on τ? **right plot**





4) The figure below shows Gaussian (blue and purple) and Lorentzian (red and green) line profiles for a very strong line ($\tau >> 1$).



a. Which type of profile dominates?

*A **Lorentzian profile dominates** for very strong lines.*

b. For this strong absorption line, does W_λ depend *more strongly* on optical depth for the Gaussian or for the Lorentzian line profile?

W_λ depends *more strongly* on τ for the **Lorentzian profile.**

5) Let's put it these ideas together to make a curve of growth.

a. **Weak lines:** The equivalent width W_λ scales linearly with optical depth τ. Using your answer to #1, how does W_λ scale with number density n?

$$\boldsymbol{W_\lambda \propto n}$$

b. **Strong lines:** *Line saturation* occurs when an absorption line becomes optically thick and begins to "bottom out." Based on your answer to #3, what happens when an absorption line begins to saturate: does the scaling between W_λ and n get stronger or weaker than in #5a?

W_λ depends less strongly on n than in #5a.

c. **Very strong lines:** What happens when the line profile from #4 begins to dominate: does the scaling between W_λ and n get stronger or weaker than in #5b?

W_λ depends more strongly on n than in #5b.

d. Use your answers from parts #5a-5c to sketch a curve of growth.