Ay 123 Lecture XIII Binaries & Modeling Stellar Evolution

Binaries and higher multiplicity is common

- $\sim 50\%$ of $1 M_\odot$ stars have at least one companion
- $\sim 100\%$ of $M \gtrsim 10 M_\odot$ stars
- Orbital periods on main sequence range from ~12 hours to $\sim 10^6$ yr
- effects on stellar evolution small if $a \gg R$

Interacting Binaries

- Stars will definitely interact if $R_1 > a$, ie one star orbits inside surface of the other
- Interaction becomes important even if $R_1 \lesssim a$ due to tidal forces

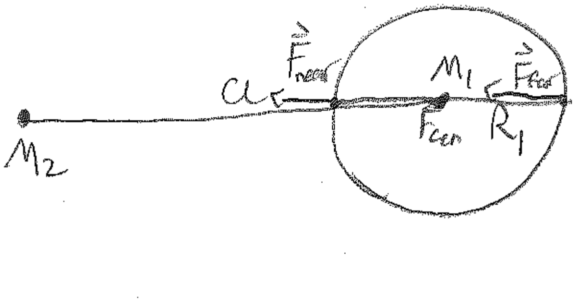
Tidal Forces

Tidal force is the difference in gravitational force of companion across the diameter of a star

$$F_{cen} = \frac{G M_2}{a^2}$$

$$F_{near} = \frac{G M_2}{(a-R_1)^2}$$

$$F_{far} = \frac{G M_2}{(a+R_1)^2}$$



For $R_1 \ll a$,

$$F_{near} \simeq \frac{GM_2}{a^2}\left(1+\frac{2R_1}{a}\right) \simeq F_{cen}\left(1+\frac{2R_1}{a}\right)$$

$$F_{far} \simeq F_{cen}\left(1-\frac{2R_1}{a}\right)$$

So the tidal force is the difference,

$$F_{tide} = F_{near} - F_{far} \simeq F_{cen}\frac{4R_1}{a} = \frac{4GM_2R_1}{a^3}$$

Tidal forces will rip the star apart if $F_{tide} \gtrsim F_g$

$$\Rightarrow \frac{4GM_2R_1}{a^3} \gtrsim \frac{GM_1}{R_1^2}$$

$$\Rightarrow a \lesssim \left(\frac{4M_2}{M_1}\right)^{1/3} R_1$$

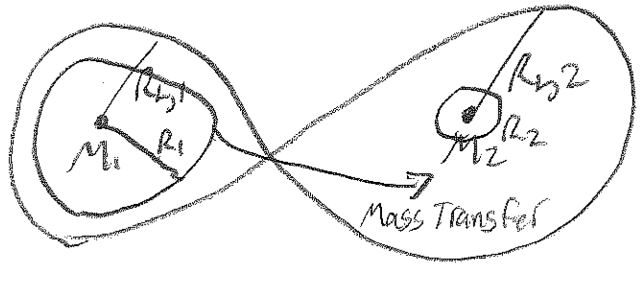
More detailed calculations yield a maximum radius R_1, ie, a Roche Lobe radius

$$R_L \simeq 0.4\left(\frac{M_1}{M_1+M_2}\right)^{1/3} a$$

For $R_1 < R_L$, material is bound to M_1. For $R_1 > R_L$, mass is lost from M_1 and accreted onto M_2 through Roche Lobe Overflow

Note

$$\frac{R_{L,1}}{R_{L,2}} \simeq \left(\frac{M_1}{M_2}\right)^{1/3}$$



So the star that donates mass first is the one with

$$\frac{R_{donor}}{R_{L,donor}} > \frac{R_{accretor}}{R_{L,accretor}}$$

$$\Rightarrow \frac{R_{donor}}{R_{accretor}} \gtrsim \left(\frac{M_{donor}}{M_{accretor}}\right)^{1/3}$$

$$\Rightarrow \rho_{donor} \lesssim \rho_{accretor}$$

This works for $M_1 \sim M_2$, but see Eggleton 2006 for extreme mass ratio systems,

Mass Transfer

When a star fills its Roche Lobe, mass transfer can be stable or unstable

-Stability criterion is

$$\frac{dR}{dM} < \frac{dR_L}{dM} \quad \text{for stable mass tranfer}$$

Change in semi-major axis:

-Assume no mass lost from system, and assume orbital angular momentum is constant:

$$J_{orb} = \left(\frac{Ga}{M_1+M_2}\right)^{1/2} M_1 M_2$$

$$\Rightarrow \frac{1}{J}\frac{dJ}{dt} = \frac{1}{2}\frac{\dot{a}}{a} + \left(1-\frac{M_1}{M_2}\right)\frac{\dot{M}_1}{M_1} = 0$$

$$\Rightarrow \frac{\dot{a}}{a} = 2\left(\frac{M_1}{M_2}-1\right)\frac{\dot{M}_1}{M_1}$$

-Mass transfer from M_1 to M_2 means $\dot{M}_1 < 0$.

-Orbital expansion occurs if $\frac{M_1}{M_2} < 1$, ie, mass transfer from less massive to more massive star. This promotes stability

Change in radius of donor:

- Convective stars and white dwarfs are $\gamma = 5/3$ polytropes,
  $R \propto M^{-1/3}$
  $\Rightarrow$ Radius increases with mass loss, promotes instability
- For radiative stars, R typically decreases as mass is lost, promoting stability

Results

- Mass transfer usually stable for radiative stars on main sequence
- Mass transfer often unstable for post-main sequence stars that develop convective envelopes
- Mass transfer often unstable if low-mass companion (e.g., planet or brown dwarf) accretes from more massive donor star.

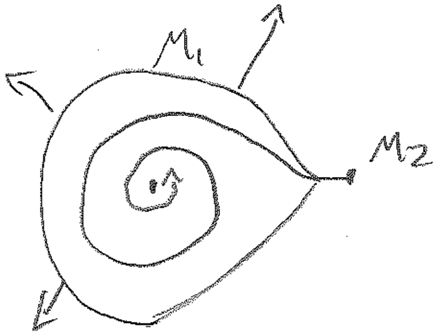
Unstable Mass Transfer

- Results in stellar merger or common-envelope event
- Binary can emerge from common envelope as two distinct stars if orbital energy can be used to eject the envelope of the donor star, leaving behind its compact core
- Can occur for red giant donor stars

α-prescription for common-envelope resut

Compare binding energy of stellar envelope

$$E_{bind} = \frac{G M_{core} M_{env}}{\lambda R_1}$$



to change in orbital energy of binary

$$E_{orb,initial} = \frac{G M_1 M_2}{2 a_{initial}}$$

$$E_{orb,final} = \frac{G M_{core} M_2}{2 a_{final}}$$

Final orbital separation is determined assuming some fraction, α, of change in orbital energy, is used to eject envelope

$M_1 = M_{core} + M_{env}$

$$\alpha \Delta E_{orb} = E_{bind}$$

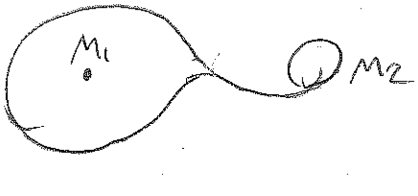
$$\Rightarrow \alpha \left(\frac{G M_{core} M_2}{2 a_f} - \frac{G M_1 M_2}{2 a_i} \right) = \frac{G M_{core} M_{env}}{\lambda R_1}$$

where λ determined from stellar structure, α determined from simulations or observations

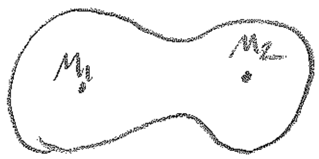
For $M_2 < M_{env}$, common-envelope can result in binary with $a_f \ll a_i$, ie, a compact binary with short orbital period

Types of mass-tranferring binaries

- Semi-detached binary
  - Stable mass transfer

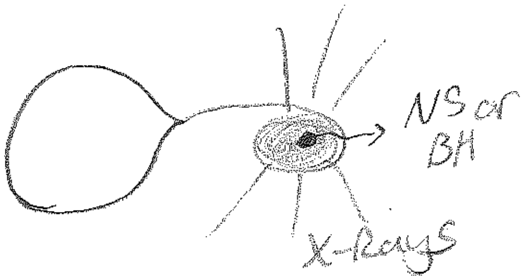


- Contact binary
  - Both stars fill Roche Lobes
  - $P_{orb} \sim 1$ days



- Cataclysmic Variables
  - White dwarfs accreting from M/K Dwarf
  - $P_{orb} \sim 1-4$ hours
  - Outbursts from variable accretion from disk
  - Polars/Intermediate Polars have magnetic white dwarf accretors → B-field truncates accretion disk
  - Post common-envelope binaries

- X-ray binaries
  - Neutron Star or black holes accreting from stellar companion
  - Low-mass XRBs accrete via Roche Lobe overflow from low-mass star
  - High-mass XRBs accrete stellar wind of massive companion star



- Millisecond pulsars
  - Outcome of Low-mass XRBs
  - Neutron star accretes mass, angular momentum, increases spin frequency to ~2ms periods

- Type Ia SNe
  - Explosions of white dwarfs when carbon-burning triggers detonation
  - Formed in binaries, two most common scenarios
    - Single-degenerate: WD accretes until $M > M_{ch}$, collapse triggers explosive C-burning
    - Double-degenerate: WD merger ignites C-burning

- Helium white dwarfs
    - Can't form from single stars MS lifetime longer than age of universe
    - Post-common envelope binary from red giant primary

- Subdwarf B stars
    - Nearly pure Helium stars, burning He in core
    - Form similar to He WDs, but common-envelope must happen near tip of RGB so that core is massive enough to ignitie He burning

- Wolf-Rayet stars
    - Massive versions of sdB stars,
    - ~10 $M_\odot$ He-burning He stars
    - Formed via mass transfer in high-mass binaries

- Merging BH/NS likely form via double common-envelope interaction