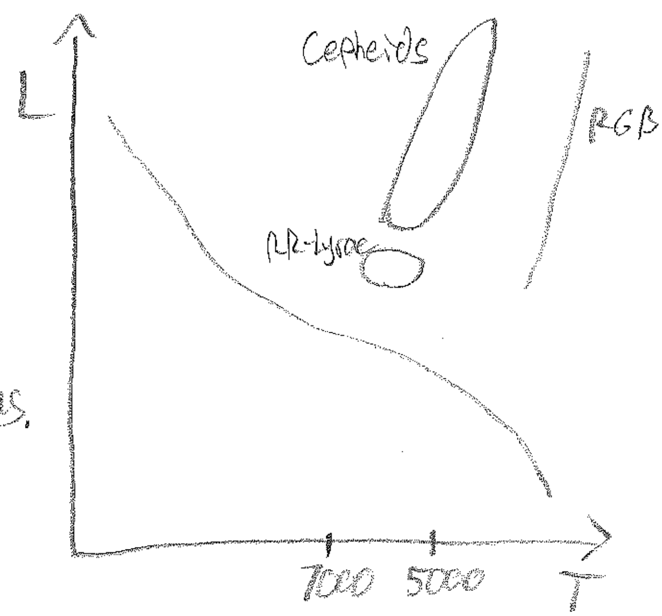
Ay 123 Lecture VIII Pulsating Stars

Some stars are <u>not</u> in hydrostatic equilibrium and show large amplitude pulsations. Pulsating stars have been observed for centuries.

Types of pulsators:

Cepheids: Post-main sequence, $M \gtrsim 5 M_\odot$ stars
- Pulsation periods of ~3-50 days, amplitudes ~ 100%
- Pulsation period-luminosity relation allows us to measure distances

RR Lyrae: He-burning, low-metallicity, low-mass ($M \lesssim M_\odot$) stars
- Pulsation period ~0.5 days
- All have similar brightness



Both appear in instability trip wit $T_{eff} \sim 6000$ K, in HR-gap.

These stars exhibit <u>radial</u> pulsations.

Adiabatic Radial Pulsations

$$\frac{\Delta P}{P} = \gamma \frac{\Delta \rho}{\rho} \quad , \quad \frac{\Delta T}{T} = (\gamma - 1) \frac{\Delta \rho}{\rho}$$

To get sense of radial pulsation mode, let's consider a radial "breathing" mode where the entire star expands and contracts.

Density

$$\bar{\rho} = \frac{3M}{4\pi R^3}$$

Perturb

$$\Rightarrow \Delta\rho = \frac{3M}{4\pi}\Delta(R^{-3}) = \frac{3M}{4\pi}\left(-3R^{-4}\Delta R\right)$$

$$\Rightarrow \frac{\Delta\rho}{\rho} = -3\frac{\Delta R}{R}$$

And $$\frac{\Delta P}{\bar{P}} = -3\gamma\frac{\Delta R}{R}$$

Radial equation of motion:

$$\Delta\left(\rho\frac{d^2}{dt^2}R\right) = \Delta\left(\frac{-dP}{dr} - \rho g\right)$$

$$\Rightarrow \bar{\rho}\,\Delta\ddot{R} \simeq -\Delta\left(\frac{\bar{P}}{R}\right) - \Delta(\bar{\rho}\bar{g})$$

$$\simeq \frac{-\Delta P}{R} + \frac{\bar{P}}{R}\frac{\Delta R}{R} - \bar{g}\Delta\rho - \bar{\rho}\Delta g$$

$$\Rightarrow \frac{\Delta\ddot{R}}{R} \simeq \frac{-\Delta P}{\bar{P}}\frac{\bar{P}}{\bar{\rho}R^2} + \frac{\bar{P}}{\bar{\rho}R^2}\frac{\Delta R}{R} - \frac{\bar{g}}{R}\frac{\Delta\rho}{\rho} - \frac{1}{R}\Delta g$$

Recall $$\bar{g} = \frac{GM}{R^2} \Rightarrow \frac{\Delta g}{\bar{g}} = -2\frac{\Delta R}{R}$$

Plugging in from above,

$$\frac{\Delta\ddot{R}}{R} \simeq 3\gamma\frac{\bar{P}}{\bar{\rho}R^2}\frac{\Delta R}{R} + \frac{\bar{P}}{\bar{\rho}R^2}\frac{\Delta R}{R} + 3\frac{\bar{g}}{R}\frac{\Delta R}{R} + \frac{2\bar{g}}{R}\frac{\Delta R}{R}$$

Now, our "equilibrium" state has $\bar{g} = \frac{GM}{R^2}$,

$$\frac{\bar{P}}{R} = -\bar{\rho}\bar{g} \Rightarrow \frac{\bar{P}}{\bar{\rho}R} = -\bar{g} = -\frac{GM}{R^2}$$

So we have

$$\frac{\Delta \ddot{R}}{R} = \left(-3\gamma + 4\right)\frac{GM}{R^3}\frac{\Delta R}{R}$$

For periodic pulsations $\Delta R \propto e^{\pm i\omega t}$

$$\Rightarrow \Delta \ddot{R} = -\omega^2 \Delta R$$

So

$$-\omega^2 = (-3\gamma + 4)\frac{GM}{R^3}$$

$$\Rightarrow \boxed{\omega^2 = (3\gamma - 4)\frac{GM}{R^3}}$$

If $\gamma < 4/3$, ω^2 is negative, ω is imaginary.
Perturbations grow exponentially $\Rightarrow$ Unstable!
In general,

$$\omega^2 \sim \frac{GM}{R^3} \sim \tau_{dyn}^{-2} \sim \sqrt{G\bar{\rho}}$$

Breathing modes have frequencies similar to the star's dynamical time. Measuring ω gives measurement of $\bar{\rho}^{1/2}$.

Exactly solving oscillation equations requires integrating an ODE. Yields similar result. Note that $L \propto R^2 T^4$ ↳ Nearly constant for stars in instability strip

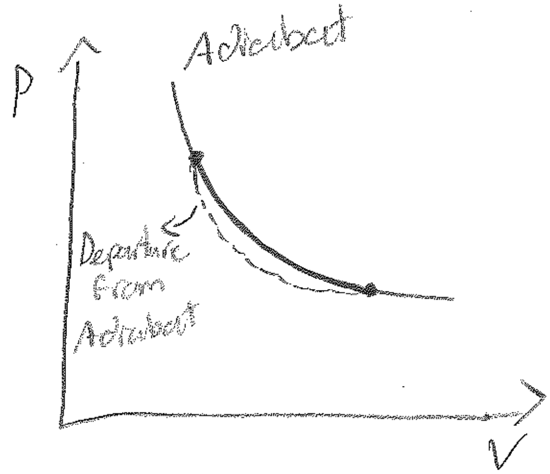
So $P_{pulse} \propto R^{3/2} M^{-1/2} \propto \frac{L^{3/4}}{T^3 M^{1/2}} \Rightarrow \boxed{L^{3/4} \propto P_{pulse}}$

↳ Weak Dependence

Period-Luminosity Relation

Excitation of Pulsations

Thus far we have assumed pulsations are adiabatic, but an adiabatic pulsation does not grow. Small departures from adiabaticity allow pulsations to grow on thermal time scale.



Heat Engines

1st Law of Thermo:

$$dU = dQ - dW$$

dU → internal energy, dQ → heat, dW → work done

$$= TdS - dW$$

For adiabatic pulsations, $dS = 0$. For a pulsation, $\oint dU = 0$ over one pulsation cycle. So, for adiabatic pulsation

$$\oint dW = \oint PdV = 0$$

→ Work Integral

For non-adiabatic pulsation,

$$\oint dW = \oint TdS = \int \frac{dQ}{dt} dt$$

Consider a pulsation that is nearly adiabatic. Then

$$\oint \frac{dS}{dt} dt \simeq 0 \simeq \oint \frac{1}{T} \frac{dQ}{dt} dt$$

Replacing $T \to T_0 + \Delta T(t)$,

$$0 \simeq \oint \frac{1}{T_0 + \Delta T} \frac{dQ}{dt} dt \simeq \oint dt \frac{1}{T_0} \left(1 - \frac{\Delta T}{T_0}\right) \frac{dQ}{dt}$$

$$\Rightarrow \quad \frac{1}{T_0}\int dt \frac{dQ}{dt} = \frac{1}{T_0}\oint dt \frac{\Delta T}{T_0}\frac{dQ}{dt}$$

$$\Rightarrow \quad W = \oint dt \frac{\Delta T}{T_0}\frac{dQ}{dt}$$

This is true for each fluid element. The work integral for a pulsation mode requires integrating over all fluid elements.

So

$$W = \oint dt \int \frac{\Delta T}{T}\frac{dQ}{dt}\, dm$$

($\frac{dQ}{dt}$ → Heat per unit mass)

Work Integral

Heat transfer occurs due to radiative diffusion in and out of fluid elements

$$\vec{F}_{rad} = -\frac{4ac\,T^3\vec{\nabla}T}{3\kappa\rho}$$

↳ $\frac{erg}{s\,cm^2}$

$$\Rightarrow \quad \frac{dQ}{dt} = -\frac{1}{\rho}\vec{\nabla}\cdot\vec{F}_{rad}$$

The perturbed heat flux is

$$\Delta\frac{dQ}{dt} = -\Delta\left(\frac{1}{\rho}\vec{\nabla}\cdot\vec{F}_{rad}\right)$$

So the work done per unit time is

$$\boxed{\frac{dE}{dt} = \frac{dW}{dt} = -\int \frac{\Delta T}{T}\,\Delta\left(\frac{1}{\rho}\vec{\nabla}\cdot\vec{F}_{rad}\right) dm}$$

Kappa Mechanism

Pulsations perturb the temperature and therefore the opacity.

Let $\kappa = \kappa_0 \rho^a T^b$

For Kramer's opacity, $a = 1$, $b = -3.5$

For H^- opacity, $a \sim 0.5$, $b \sim 9$

Then the perturbation in opacity is

$$\frac{\Delta \kappa}{\kappa} = a \frac{\Delta \rho}{\rho} + b \frac{\Delta T}{T}$$

$$\kappa_\rho = \left(\frac{\partial \ln \kappa}{\partial \ln \rho}\right)_T \qquad \left(\frac{\partial \ln \kappa}{\partial \ln T}\right)_\rho = \kappa_T$$

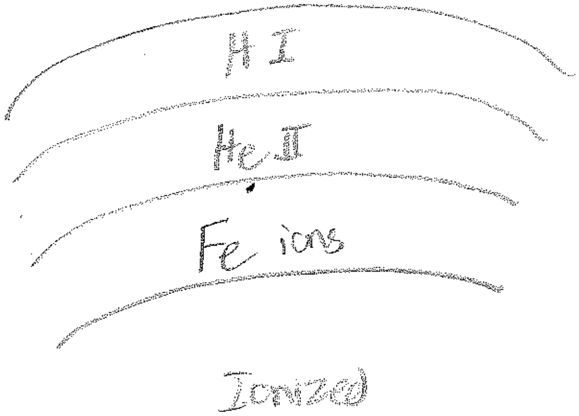
For nearly adiabatic pulsation, $\frac{\Delta T}{T} \simeq (\gamma - 1) \frac{\Delta \rho}{\rho}$

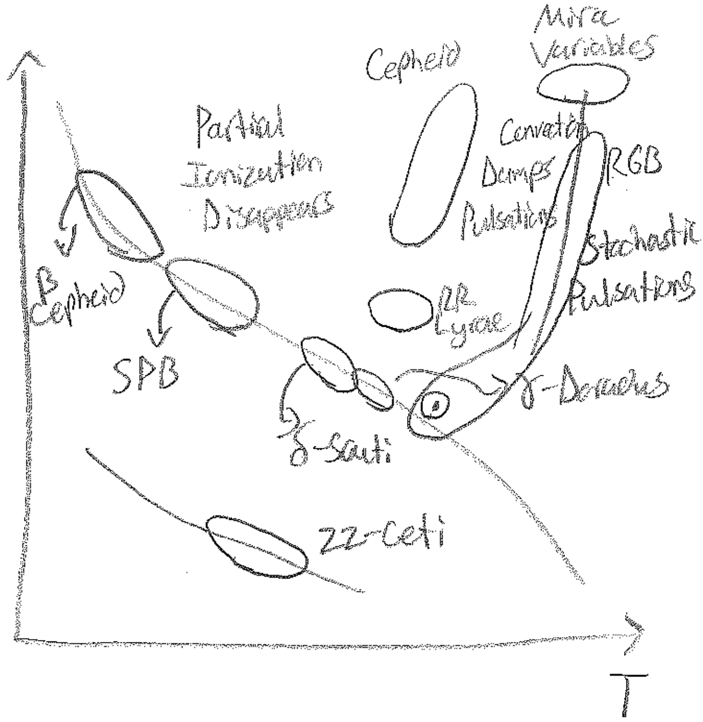
$$\Rightarrow \frac{\Delta \kappa}{\kappa} \simeq \left[a + b(\gamma - 1)\right] \frac{\Delta \rho}{\rho}$$

For Kramer's opacity, $\frac{\Delta \kappa}{\kappa} \propto -\frac{\Delta \rho}{\rho}$

Compression $\Rightarrow$ increase ρ and T, decrease κ
increase heat flux, element cools,
pulsation decays

In most parts of stars, pulsations are damped. In partial ionization zones, however, $\gamma \to 1$, and we can have κ increase under compression. Thermal energy is stored in the fluid element, and the pulsation grows. For unstable pulsations to occur, driving in partial ionization regions must outweigh damping in other regions.


H I
He II
Fe ions
Ionized


Mira Variables
Cepheid
Partial Ionization Disappears
Convection Damps Pulsations
RGB
Stochastic Pulsations
β Cepheid
SPB
RR Lyrae
δ-Scuti
γ-Doradus
ZZ-Ceti
T