Ay 123 Lecture 1

What is definition of a star?

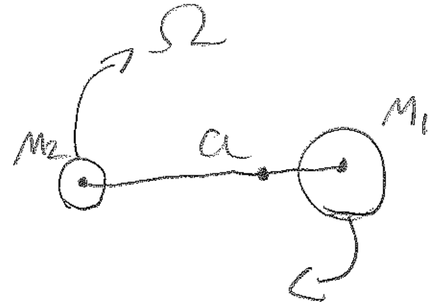
- bound by self-gravity
- supported by internal pressure
- radiates energy from internal energy source, usually thermonuclear fusion

Which quantities characterize a star?

- mass M
- radius R
- what is important timescale?
  Gravity is important, so $t_{dyn} = \sqrt{R^3/GM}$
  is natural timescale, known as dynamical timescale of star
- other important properties
  - Composition, e.g., metallicity Z or mean molecular weight μ
  - Age, evolutionary state
  - Luminosity L
  - Rotation rate Ω
  - Surface temperature T_{eff}
  - Binarity, are there companion stars and/or planets

All are related. Dimensional analysis is key!

1.1 Stellar Masses



Kepler's Law:

$$\Omega^2 = \frac{G(M_1+M_2)}{a^3}$$

Ω ↳ Orbital Frequency

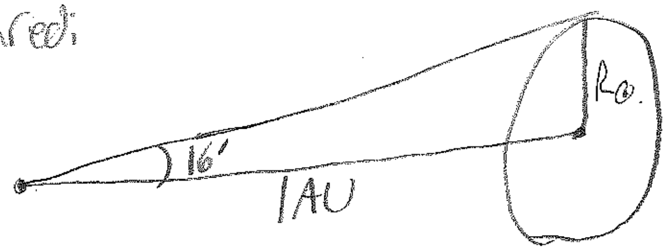
a^3 → semi-major axis

Measuring Earth's orbital frequency is easy! Finding distance to Sun is harder. Turns out that Earth's semi-major axis is $1AU = 1.5\times10^{13}$ cm. Solving for M_{sun},

$$\boxed{M_\odot = 1.99\times10^{33}\,g}$$

Sun's radius can also be measured:

$$\boxed{R_\odot = 6.955\times10^{10}\,cm}$$



and

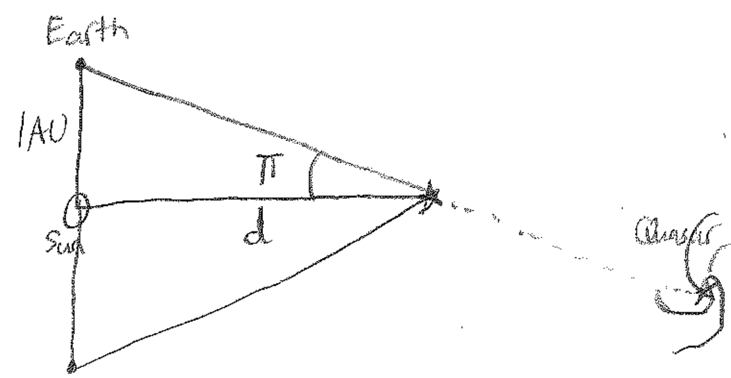
$$t_{dyn} = \sqrt{R^3/GM} = 1.6\times10^3\,s$$

How do we measure distances to other stars? Parallax

$$\tan\pi = \frac{1AU}{d} \simeq \pi$$

$$d = \frac{1AU}{\pi}$$

↳ measured in radians



There are $180^\circ = 1$ radian

$60' = 1^\circ$

$60'' = 1'$

$\Rightarrow 206265' = 1$ radian

So

$$d = \frac{1 AU \times 206265}{\pi(')} = \frac{1 pc}{\pi(')}$$

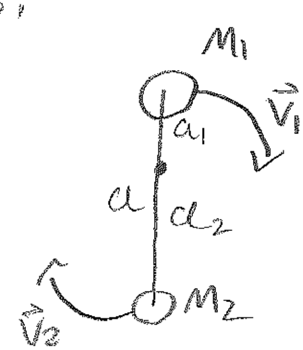
$$1 pc \simeq 206265 \, AU \simeq 3.1 \times 10^{18} cm \simeq 3.3 \, lyr$$

Gaia satellite can measure parallaxes to $\sim 10\,\mu as$ precision. This means we can measure stellar distance out to almost 10^5 pc, i.e., across entire galaxy. We will soon have distances, precise stellar properties to over 10^9 stars (compared to $\sim 10^5$ currently).

How can we measure masses of other stars?

Eclipsing binaries

Measure radial velocities



For circular orbits,

$$V_1 = \Omega a_1 \sin i$$

$$V_2 = \Omega a_2 \sin i$$

$$\Omega^2 = \frac{G(M_1 + M_2)}{(a_1 + a_2)^3}$$

$$M_1 a_1 = M_2 a_2$$

Combine:

$$M_1 + M_2 = \frac{(v_1+v_2)^3}{G\Omega \sin^3 i} \qquad \frac{M_1}{M_2} = \frac{v_2}{v_1}$$

For eclipsing binaries, $i \simeq \pi/2$.

Observationally, we find stars with $10^{-1} M_\odot \lesssim M \lesssim 10^2 M_\odot$

Lower limit is set by minimum core temperature needed to sustain hydrogen fusion.

Upper limit not totally understood but related to Eddington limit.

1.2 Stellar Radii

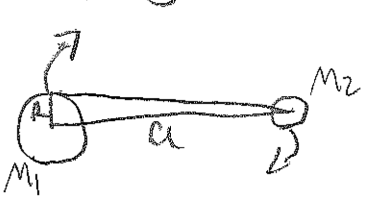
Stars don't have a precise surface, but typically all their light emerges from a narrow photosphere with height much less than distance to center of star, so photospheric radius is fairly well-defined.

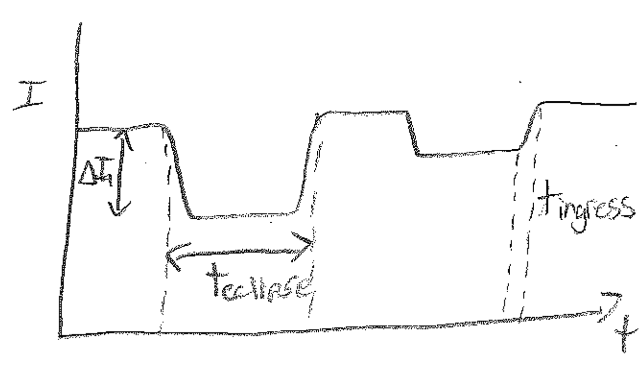
Angular radii of other stars:

$$\Theta \approx \frac{R}{d} \simeq \frac{R_\odot}{1\,pc} \simeq 2 \times 10^{-8} \quad \text{for nearby stars}$$

Not resolvable even from space, except using interferometry, and this can only reach nearby stars. Giant stars have $R \gtrsim 100 R_\odot$, but are rare and typically have $d \gtrsim 100\,pc$

Best method: Eclipsing binaries





Duration of eclipse is

$$\Delta t_{eclipse} \simeq P_{orb} \frac{2R}{a}$$

Need masses from radial velocities

$$\Rightarrow R \simeq \frac{a}{2} \frac{\Delta t_{eclipse}}{P_{orb}}$$

$$t_{ingress} \simeq R_2/R_1$$

Relative radii and luminosities also measured from eclipse depths

Double-lined eclipsing binaries yield accurate values of M_1, M_2, R_1, R_2, a by measuring P_{orb}, v_1, v_2, $t_{eclipse}$, $t_{ingress}$.

Observationally, we find stellar radii

$$10^{-2} R_\odot \lesssim R \lesssim 10^{3} R_\odot$$

White Dwarfs ($10^{-2} R_\odot$), Red Supergiants ($10^{3} R_\odot$)

Neutron star radii have not been precisely measured but have been constrained to $R_{NS} \sim 12 \pm 2$ km.

1.3 Stellar Temperatures

Stars have large core temperatures $T \gtrsim 10^7$ K

They radiate energy at surface at much lower temperature.

Stars are approximately blackbodies, and their radiated luminosity is approximately that of a spherical blackbody:

$$\boxed{L = 4\pi \sigma_B R^2 T_{eff}^4}$$

σ_B: Stefan-Boltzmann constant

$$\sigma_B = \frac{2\pi^5 k_B^4}{15 c^2 h^3} \simeq 5.67 \times 10^{-5} \frac{erg}{s\,cm^2 K^4}$$

Important: stars are not perfect blackbodies and deviations (e.g., spectral absorption lines) allow us to measure stellar properties. Also, we are usually not able to measure star's total luminosity, but only energy radiated at certain (usually visual band) wavelengths. A bolometric correction must be applied to obtain the total luminosity.

Planck's blackbody radiation law is

$$B_\nu(T) = \frac{\partial I_\nu}{\partial \nu} = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

Note $\lambda \nu = c$

$$\Rightarrow d\nu = -\frac{c}{\lambda^2} d\lambda$$

So

$$B_\lambda(T) = \frac{\partial I_\lambda}{\partial \lambda} = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

Wien's Law: Wavelength where spectral energy distribution peaks can be found by setting $\partial B_\lambda / \partial \lambda = 0$. Result is

$$\lambda_{max} = \frac{0.29\,\text{cm K}}{T}$$

$$\Rightarrow \lambda_{max} \simeq 500\text{ nm} \left(T/T_\odot\right)^{-1}$$

where the Sun's effective temperature is $\boxed{T_\odot = 5770\text{ K}}$

This is in the middle of the visual spectrum. This is why we see in visual wavelengths.

Stellar Classification

Stellar spectral types based mainly on strength of hydrogen Balmer absorption lines

O B A F G K M (L T Y)

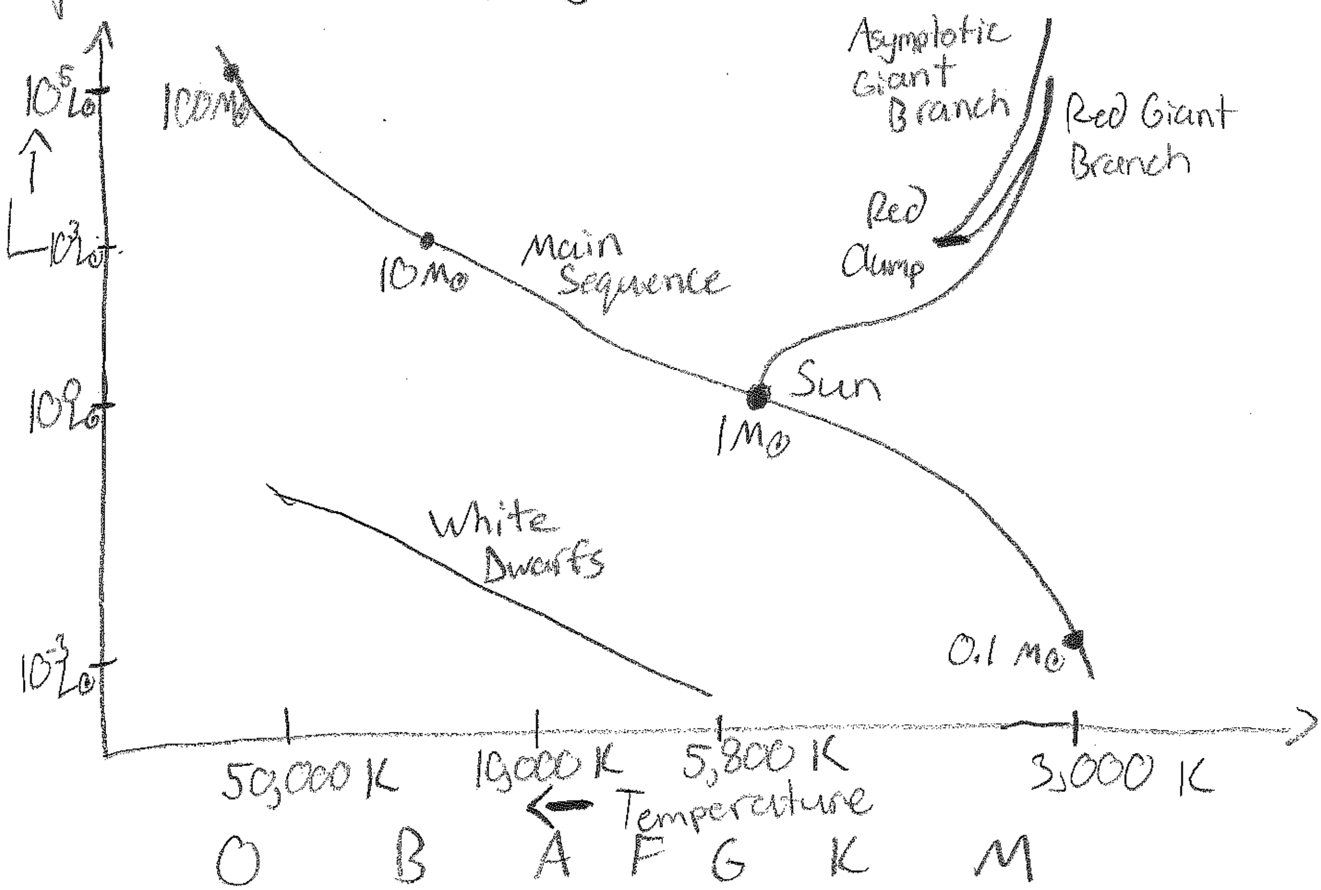
weakest Balmer lines

↳ Strongest Balmer lines

Energy needed to move electron from n=2 to n=1

Stars with surface temperature $kT \sim 2\,eV$ produce strongest Balmer lines. This translates to $T \sim 20{,}000\,K$ = B type

So spectral sequence can be translated into temperature sequence, or color sequence (hotter = bluer). If a star's absolute luminosity can be measured, we can put in a Hertzsprung-Russell diagram:



Stellar Evolution

Stars on main sequence burn hydrogen into helium. They spend ~90% of their lives there.

Low-mass stars ($M \lesssim 2.5\ M_\odot$) evolve up red giant branch as they burn hydrogen in a shell around a helium core. Eventually the helium core becomes hot enough to ignite helium fusion, and star settles onto red clump. After helium is exhausted, stars move up asymptotic giant branch as they do helium shell burning. These stars are not massive enough to ignite carbon burning, and become carbon-oxygen white dwarfs

Stars with ($2.5\ M_\odot \lesssim M \lesssim 5 M_\odot$) evolve similarly, but do not have to evolve all the way up red giant branch before igniting helium.

Stars with ($5 M_\odot \lesssim M \lesssim 8\ M_\odot$) are massive enough to trigger carbon burning, but not neon burning. They become oxygen-neon white dwarfs.

Stars with $M \gtrsim 8\ M_\odot$ are massive enough to trigger neon and silicon burning, developing iron cores. Fusion becomes endothermic for elements heavier than iron. So if these cores collapse, yielding neutron stars or black holes, and producing supernovae!

Low-mass stars with $M \lesssim 0.8 M_\odot$ do not evolve off the main sequence within the age of the universe.