Ay123 Problem Set 5

Due Tuesday, December 10, 5pm

1. **Cepheid Variable** (10 points)

   The mass and mean radius of a typical Cepheid variable are $\log(M/M_\odot) = 0.8$ and $\log(R/R_\odot) = 1.4$.

   (a) Use the continuity equation to show that a radial perturbation that satisfies $\Delta\rho/\rho = -\Delta V/V$ (where V is volume) implies that

   $$\frac{\partial}{\partial r}\frac{\Delta r}{r} = 0\,, \tag{1}$$

   where Δr is the radial Langrangian displacement.

   (b) For a radial pulsation satisfying equation 1, use the continuity equation to relate $\Delta\rho/\rho$ to $\Delta r/r$.

   (c) Use this relation in the momentum equation to show that $\omega^2 = (3\gamma - 4)g/r$. What does this imply about the stability of the star when $\gamma < 4/3$?

   (d) For $\gamma = 5/3$, derive an expression relating the luminosity of the star to its temperature, pulsation period, and surface gravity.

   (e) Using $\gamma = 5/3$, for a pulsation amplitude $\Delta r/r_0 = 0.1$, compute the fractional surface temperature perturbation $\Delta T_{\rm eff}/T_{\rm eff}$ and luminosity perturbation $\Delta L/L$.

2. **Binary Stars** (10 points)

   The minimum orbital separation of a star with mass M and radius R in a binary star system is

   $$a_{\rm min} \simeq \frac{5}{2}\left(\frac{M_{\rm tot}}{M}\right)^{1/3} R\,,$$

   where $M_{\rm tot}$ is the total mass of the binary system.

   (a) Show that the minimum orbital period of the binary is

   $$P_{\rm min} \simeq 5\pi\left(\frac{15}{8\pi}\right)^{1/2}(G\rho)^{-1/2}\,,$$

   where ρ is the average stellar density. Evaluate $P_{\rm min}$ for a binary system of two red giants with $\rho = 10^{-6}\,{\rm g/cm^3}$, two Sun-like stars with $\rho = 1\,{\rm g/cm^3}$, two white dwarfs with $\rho = 10^6\,{\rm g/cm^3}$, and two neutron stars with $\rho = 3\times10^{14}\,{\rm g/cm^3}$.

   (b) Consider a red giant of $M_1 = 1\,M_\odot$, with a core mass $M_c = 0.5\,M_\odot$, envelope mass $M_e = 0.5\,M_\odot$, and radius $R_1 = 100\,R_\odot$. It undergoes a common-envelope event with a low-mass secondary star of mass M_2 and radius R_2, which ejects the envelope of the red giant. The α prescription for common-envelope events predicts the final orbital separation a_f:

   $$\alpha\left(\frac{GM_cM_2}{2a_f} - \frac{GM_1M_2}{2a_i}\right) = \frac{GM_cM_e}{R_1}\,. \tag{2}$$

   Solve equation 2 for a_f. Show that when α is of order unity and $M_2 \ll M_e$, the final orbital separation satisfies $a_f \ll a_i$, and equation 2 reduces to

   $$a_f \simeq \frac{\alpha}{2}\frac{M_2}{M_e}R_1\,.$$

(c) A stellar merger will occur if the final separation a_f between the secondary and the primary's core is smaller than the minimum orbital separation possible for the secondary star. By replacing a_f with $a_{\min}$ for the secondary, and using $M_2 \ll M_c$, find the minimum secondary mass that can eject the envelope of the primary without merging with the core of the primary. Evaluate this mass for $\alpha = 0.5$ and typical brown dwarf radius $R_2 = 0.1 R_{\odot}$.

3. **Stellar Spectra, Part II** (15 points)

   Download and complete the Jupyter notebook problem from the course website. You should turn in a printout of your completed notebook.

4. **MESA Project, Part 2** (15 points)

   Complete the MESA Project as instructed in Homework 4.