Ay101 Problem Set 4

Due Tuesday, November 26, 5:00 pm

1. **Protostars** (10 points)

   (a) Find the average density and central temperature (as a function of mass) of an accreting protostar whose initial radius is given by the expression

   $$\frac{R}{R_\odot} = \frac{43.2}{1-0.2X}\frac{M}{M_\odot}$$

   if its structure is approximated by a $n = 1.5$ polytrope with hydrogen mass fraction $X = 0.7$ and helium fraction $Y = 0.3$.

   (b) Now suppose the star contracts to smaller radius (at constant mass) but maintains a polytropic structure until its collapse is halted when the central temperature reaches $T_{\rm crit}$ required for hydrogen burning. Show that the greater the mass of the star, the smaller the density at the point where $T_{\rm crit}$ is reached:

   $$\rho_{\rm crit} = 1.52\,\frac{1}{M^2}\left(\frac{k_B T_{\rm crit}}{\mu m_{\rm H} G}\right)^3$$

   (c) Noting the criterion for electron degeneracy, estimate the critical mass below which collapse is halted by electron degeneracy, not by hydrogen burning. After dropping factors of order unity, show that this mass is related to the Chandrasekhar limit, $M_{\rm Ch}$, by the approximate relation

   $$\frac{M_{\rm crit}}{M_{\rm Ch}} \sim \left(\frac{\mu_e}{\mu}\right)^{3/2}\left(\frac{k_B T_{\rm crit}}{m_e c^2}\right)^{3/4}$$

   Evaluate this mass for $T_{\rm crit} = 5\times10^6$ K and $M_{\rm Ch} = 1.4M_\odot$.

2. **Supernova Shock Revival from Neutrino Heating** (10 points)

   Consider the final iron core of a massive star with a mass $M_{\rm Fe} = 1.5\ M_\odot$ and radius $R_{\rm Fe} = 3\times10^8$ cm. When this core collapses, the initial collapse stops when the central core with a mass $M_{\rm core} = 0.7\ M_\odot$ reaches nuclear densities. At this density the core bounces, driving a shock with an energy $E_{\rm bounce} = 10^{51}$ erg into the infalling outer core.

   (a) Estimate the energy that is required to photodissociate 0.8 $M_\odot$ of Fe into alpha particles and neutrons. Compare this energy to the bounce shock energy and comment on the fate of the shock.

   (b) In the proto-neutron star (with an initial radius $R_{\rm PNS} = 2\times10^6$ cm), the mean free path of neutrinos is $l_\nu = 30$ cm. Estimate the diffusion time for neutrinos to escape from the proto-neutron star and hence estimate the neutrino luminosity during the initial neutron-star cooling phase.

(c) Assuming that 10% of the neutrino luminosity is absorbed by the infalling outer core, estimate how long it takes to absorb enough neutrino energy to reverse the infall of the $0.8\,M_\odot$ outer core and drive a successful supernova explosion with a typical explosion energy of 10^{51} erg. Assume the outer core has initial energy per unit mass $\epsilon = -GM_{\rm Fe}/R_{\rm Fe}$. Compare this time to the dynamical (free-fall) timescale of the proto-neutron star with initial mass $M_{\rm core}$.

(d) Once the star explodes, the duration of the observed supernova is determined by the photon diffusion time through the expanding ejecta. Provide an order of magnitude estimate of this diffusion time $t_{\rm diff}$, as a function of the ejecta mass M and the explosion kinetic energy E, assuming pure helium ejecta and electron scattering opacity. Remember that the radius of the ejecta is increasing with time as $R = vt$, where v is the ejecta velocity. The duration of the observed supernova is approximately the time at which $t = t_{\rm diff}$. Evaluate $t_{\rm diff}$ for an ejecta mass of $5\,M_\odot$ and $E = 10^{51}$ erg.

3. **Stellar Spectra, Part 1** (15 points)

   Download and complete the Jupyter notebook from the course website. You should turn in a printout of your completed notebook.

4. **MESA Project, Part 1** (15 points)

   Pick **one** of the following projects, and complete the project using MESA. The report is due with Homework 5 so nothing is due with this homework, but I advise you to start early as this MESA project will take more time to complete than prior homework questions. You should turn in a report with 1-2 pages of text plus additional pages for figures. Use logarithmic axes in your plots where appropriate. Describe some of the important parameters in MESA and why you chose the values that you used. Identify any challenges you encountered (other than installing the software). Finally, spend the majority of your report describing what you learned, paying particular attention to the prompts below. Use the MESA tutorial to get started. You can use the tutorial as a template for your own MESA runs. Most necessary modifications to your inlists can be found in the `star_job defaults` and `controls_defaults` at the MESA homepage.

   Note that MESA computes the pre-main sequence in a couple minutes. The main sequence itself takes mere seconds. The post-main sequence evolution up the red giant branch might take hours. Plan accordingly.

   Thanks to Sterl Phinney and Evan Kirby for helping construct these projects.

   (a) **Construct an Isochrone of a Globular Cluster**

   A typical globular cluster has an age of 12 Gyr and a metallicity of $Z = 0.0002$. In this project, you will use MESA to create an isochrone for such a globular cluster.

   Compute the evolution of at least ten stars starting from the pre-main sequence. Stop the models at 12 Gyr or at the onset of core helium burning (the top of the red giant branch).

   The stars must span a range of masses. One of the goals of this project is to see which stars lie on the main sequence and which stars are on the red giant branch (RGB). You also want to see the shapes of both the main sequence and the RGB in your isochrone. In order to have a well-populated RGB at 12 Gyr, you will need to have a handful of stars in a very narrow mass range. From my own experimentation, the mass range is 0.810–0.818 $M_\odot$. This range is based on assuming the default values of all parameters except mass and metallicity in MESA v. 10108. At least five of your stars should lie somewhere in this mass range. The rest of your stars should be less massive. A good number for the lowest mass star would be something like 0.4 $M_\odot$.

   Your report should address why I advised you to put many of your stars in the narrow mass range specified above. You should also address why I did not advise you to include stars more massive than 0.818 $M_\odot$.

   Plot all of the stars on the same Hertzsprung–Russell diagram at the age of 12 Gyr. Respect the observational conventions of the diagram (the direction of each axis). Make sure that your most massive stars reached 12 Gyr and did not terminate earlier than that.[1] The line connecting the stars is called the isochrone. Also plot the evolutionary track of the most massive star. You might also like to include the evolutionary tracks of other representative stars. Does the track go through the isochrone? Why or why not?

(b) **Stellar Structure Along the Main Sequence**

Construct ZAMS models of stars with the solar composition at 0.3, 1.0, and 3.0 $M_\odot$. Show the temperature–density (T–ρ) diagram for each star, possibly on the same diagram with each star clearly labeled. Discuss any similarities or differences you observe among the three stars.

Also plot opacity as a function of mass coordinate or radius. Where is the opacity high? What are the dominant sources of opacity for various locations within the stars? On the same diagram, show where the star is radiative and where it is convective. Can you relate the opacity to the heat transport? Is it always true that high opacity corresponds to one of the two methods of heat transport? Is it always true that convection/radiation corresponds to high/low opacity? Why or why not?

Finally, repeat this exercise with a star with 1/100 of the solar metallicity. How can you explain the changes that you observe?

(c) **Energy Generation**

Compare the power output of the *pp* and CNO cycles on the main sequence for 0.5 $M_\odot$, 1 $M_\odot$, and 2 $M_\odot$ stars.

What causes the star to leave the main sequence and start its ascent on the red giant branch? I'm looking for an answer somewhat deeper than "it runs out of hydrogen in the core." You may like to read up on the *Schönberg–Chandrasekhar limit* described on page 137 of HKT (2^{nd} edition). Make some plots based on the MESA output to support your answer.

Then, pick one of these models and show that hydrogen burning happens in a thin shell on the red giant branch. Plot the evolution along the red giant branch of the location of the hydrogen-burning shell in both mass coordinate and radius. Give a narrative interpretation of these diagrams.

(d) **Massive Stars**

Construct and run a model of a 15 $M_\odot$ star, starting from the `mesa/star/work` directory. In `inlist_pgstar`, add the command `Abundance_win_flag = .true.` so that MESA plots the internal chemical composition of the star. Disable the stopping conditions in `inlist_project` and let the model run as long as it can. It should be able to run nearly until the core collapses. Watch the plots as it runs and try to figure out what is going on.

List the sequence of primary nuclear burning reactions that occur at the center of the star, starting at the main sequence, and ending with the development of an iron core. Are the burning regions typically convective or radiative, and why?

While the star is burning carbon in its core, make a plot of density and temperature as a function of radius. You will need to use logarithmic axes. Label the carbon burning and helium burning zones, and the convective envelope.

Does the star significantly change its luminosity, radius, or surface temperature after the start of carbon burning in the core? Explain why or why not. Does the core density or temperature change very much? Explain why or why not.

What element is most abundant in the core of the star just before it collapses? How much of the star's mass is in this core? Explain this value. You'll notice some helium in the core of the star just before it collapses. Why is helium starting to appear?

(e) **Something Completely Different**

Invent your own project, but clear it with Jim first. There are dozens of projects listed at http://cococubed.asu.edu/mesa_market/education.html.

[1]My 0.818 $M_\odot$ model reached the He core flash at 11.995 Gyr.