Ay101 Problem Set 1

Due date Monday, Oct 14

1. **The scale of the Sun** (10 points)

   Measuring the scale of astronomical objects is not easy. In this problem, you may use relevant physical laws and constants (e.g., G, c and σ_B), but do not look up measurements of the Sun or solar system.

   (a) Venus has an orbital period of 225 days. Using Kepler's laws, what is its semi-major axis in units of AU?

   (b) At conjunction with the Sun, astronomers on Earth emit a radio pulse directed toward Venus. It takes 276 seconds to detected the radio waves that reflect off Venus. Assuming circular orbits for the Earth and Venus, use this information to compute the length of 1 AU in cgs units.

   (c) Use your results above to compute the mass of the Sun in cgs units. Using its measured angular radius of 16 arcmin, compute the radius of the Sun in cgs units.

   (d) The surface temperature of the Sun is $T = 5770$ K. Using the blackbody radiation law, compute the luminosity of the Sun in cgs units.

2. **Stars are gases** (10 points)

   The average density of the Sun is slightly larger than that of water.

   (a) Provide a quantitative relation between the temperature and density of a star which indicates when we can treat it as a gas (as opposed to liquid or solid) throughout its interior, in spite of the very high densities. Is our assumption valid at the center of the Sun, where the density is about 100 times the average density? Note that your answer should only involve classical physics, no quantum mechanics.

   (b) If all stars have roughly the same central temperature (that of the Sun), use a scaling argument to determine the stellar mass at the which the simple non-interacting ideal gas assumption breaks down.

3. **A toy star** (15 points)

   Assume that a star obeys a linear density model:

   $$\rho(r) = \rho_c \left(1 - \frac{r}{R}\right),$$

   where ρ_c is the central density and R is the radius of the star.

   (a) Find an expression for the central density in terms of R and the total mass M.

   (b) Use the equation of hydrostatic equilibrium and zero boundary conditions to find the pressure as a function of radius. Your answer will be in the form $P(r) = P_c \times f(r/R)$, where $f(x)$ is a function that you will determine. What is the dependence of the central pressure P_c in terms of M and R?

   (c) What is the central temperature T_c assuming the ideal gas equation of state? How does it scale with mean particle mass?

(d) Find the ratio of the radiation pressure to the gas pressure at the center of this star as a function of the total stellar mass, expressed in units of $M_\odot$. At what mass does radiation pressure become comparable to the ideal gas pressure?

4. **Cluster Color Magnitude Diagram** (15 points)

The goal of this problem is to make a color magnitude diagram for M67, a nearby open cluster, using *Gaia* data.

(a) Determine the location of M67 using public tools.

(b) Download a list of stars near M67 from https://gea.esac.esa.int/archive/. Make sure you download all the necessary data for the steps below (e.g., `ra, dec, parallax, pmra, pmdec, bp_rp`), and I recommend downloading a list of 2000 targets.

(c) Using python or whichever tool you prefer, plot a histogram of your stars as a function of parallax. Use this histogram to calculate a rough estimate of M67's distance. Select an appropriate parallax range to include almost all M67 stars, but exclude as many interlopers as possible.

(d) To further select M67 stars, plot the proper motions of the selected stars. Choose stars that are moving appropriately.

(e) Using your best selection criteria, make a color-magnitude diagram by plotting M_G as a function of $(BP - RP)_0$, where M_G is the absolute magnitude in the Gaia G band and $(BP - RP)_0$ is the B-R color.

Submit all three plots (parallax histogram with selection criteria shown, proper motion diagram with selection criteria shown, and CMD). Include a description of what you did for each step. Make sure to plot your data appropriately. That means making individual points easy to distinguish or, if there are too many points, making something like contours to display the data.

5. **Using the MESA stellar evolution code** (10 points)

Go to http://mesa.sourceforge.net/, and read about how to download and install MESA. You will likely want to run this on a Mac or Linux machine. I highly recommend first installing the MESA SDK, as instructed on the Mesa website. Download and install the latest public release of MESA, version 12115 as of October 4, 2019.

(a) Run the default stellar model located in mesa/star/work/. At time step 10, what is the core temperature and surface temperature of the model?

(b) Evolve a $1\,M_\odot$ model up the red giant branch.

-Change the starting mass to $1\,M_\odot$.

-You may want to disable the `create_pre_main_sequence_model` option.

-Make sure to change the stopping condition so that the model will evolve beyond core hydrogen depletion.

-What is the surface temperature and radius of the star when its luminosity reaches $50\,L_\odot$?